FLOW AND SEDIMENT TRANSPORT IN A SAND BEDDED MEANDER[1]

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ABSTRACT

The boundary shear stress pattern and the superelevation of the water surface in a meander on a small stream are predicted from two simple equations representing a frictionally dominated force balance. This comes about even though inertial forces due to local bed topography delay crossing of the boundary shear stress maximum to a position farther downstream than would otherwise be expected. Measured bedform migration rates reflect the boundary shear stress field, and in this river, bedform crest orientations respond to gradients in the shear stress field by becoming oblique to the general flow direction. When so aligned the bedforms interact with the helical flow in the curved channel and induce a near bottom secondary current which, in the upstream part of the bend, is outward above the crests and inward along the troughs. At the downstream end of the bend this pattern is reversed and significant quantities of sediment are transported along troughs from one point bar to the next. This near bottom flow pattern imposes a cross-isobath, zig-zag trajectory on the sediment grains and sorts the bed material. Further, it provides a realizable mechanism for maintenance of equilibrium channel geometry in streams with bedforms.

INTRODUCTION

Meandering rivers which transport sand primarily as bedload have beds that are deformed into bar-pool topography and usually are covered with mobile bedforms. The geometry of the bedforms is dependent on the flow field through a bend. Crest migration rates are greatest in zones of highest boundary shear stress and an asymmetrical boundary shear stress field causes the development of significant crest obliquity with respect to the main flow direction. Because bedforms substantially alter the near bottom flow field, they dictate the path of sediment movement and, therefore, are important to maintenance of the equilibrium cross sectional geometry of a meander.

This paper presents a description of the effects of dunes and sandwaves on sediment paths and sorting through a meander in Muddy Creek, Wyoming, and suggests their possible role in maintaining a stable cross sectional geometry. It is a contribution to a rapidly increasing body of knowledge derived from field studies of meandering rivers (Jackson 1975, 1976; Bridge and Jarvis 1976, 1977; Hickin and Nanson 1975; Bathurst, et al. 1977). It also demonstrates the usefulness of a model that evolved largely in the engineering literature and that, with further research, should lead to a quantitative theory of meander form. A conceptual application of aspects of this model to field studies was discussed by Jackson (1975) and a specific form of the model, developed by Engelund (1974), was used by Bridge (1977).

The model states, in essence, that the gross features of flow and sediment transport through a typical bend can be examined using a simple frictionally dominated force balance derived from the Reynolds equation of motion. This equation, for the mean velocity field in turbulent flow, can be written as follows in a Cartesian coordinate system using the Einstein convention in which a repeated index in any term implies summation over the three coordinate directions:

$$\rho \frac{\partial u_j}{\partial t} + \rho u_i \frac{\partial u_j}{\partial x_i} = \frac{\partial p}{\partial x_j} + \frac{\partial \tau_{ij}}{\partial x_i} - \rho g \cos(g, j) \tag{1}$$

Here u_k is the mean velocity component in the kth direction ($k = i$ or j), x_k is spatial coordinate in the kth direction, t represents time, ρ repre-

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sents fluid density, p represents the pressure, τ_{ij} is the total deviatoric (nonisotropic) stress acting on the ith surface in the jth direction, g is the gravitational acceleration, and cos (g, j) is the cosine of the angle between the vertical and x_j directions. The first term on the left-hand side expresses the force per unit volume due to temporal acceleration of the fluid at points fixed in space; the second term represents the inertial force per unit volume resulting from spatial rates of change in velocity. These are balanced by the pressure gradient, the stress divergence (a force per unit volume resulting from internal friction), and the gravitational force per unit volume.

Equation (1) has no known general solution but can be simplified for many important flow problems. For example, in the case where internal friction is relatively unimportant and the fluid particles do not rotate significantly so that $u_i\, \partial u_j/\partial x_i \simeq \partial(u_i^2)/\partial x_j$, equation (1) can be integrated to give the Bernoulli equation. In contrast, if frictional resistance has a much greater influence on the flow field than change in inertia, then $|\partial\tau_{ij}/\partial x_i| \gg |u_i\, \partial u_j/\partial x_i|$ so for steady flow only the right-hand terms in equation (1) remain. Integration of these terms for a system with a free surface leads to

$$\tau_{zx} = \tau_b\left(1 - \frac{z}{h}\right) \tag{2a}$$

and

$$\tau_b = -\rho g h S \tag{2b}$$

Here τ_{zx} is the shear stress on a surface parallel to the mean stream bed in the downstream direction, τ_b is the downstream component of the boundary shear stress, z is the distance above the lower boundary, h is the depth of flow, and S is the component of water surface slope in the downstream direction. In the first case the flow is said to be inertial and in the second case it is said to be frictional.

In systems for which the force balance is predominantly inertial but in which the other terms cannot be neglected the flow is said to be inertially dominated. By analogy, flow also can be frictionally dominated. In both situations the effects of relatively small, but not negligible, terms are included in the solution through a perturbation expansion. With this commonly used mathematical technique (e.g., Carrier and Pearson 1976) the equations are first solved for the predominant balance, the zero order force balance, yielding what is known as the zero order solution. Corrections required by groups of successively smaller terms in the force balance are made to this solution in a rigorously specified order. For the perturbation expansion to be valid the corrections at a given order must be significantly smaller in magnitude than the solution for the preceding order.

Most theories for flow through bends transform equation (1) into a convenient coordinate system, usually a cylindrical one, and expand it into three scalar equations. After grouping the terms according to their estimated magnitude with scaling arguments, a formal perturbation expansion is carried out, implicitly or explicitly, and equations are obtained (Rozovskii 1957; B. Yen 1972; Engelund 1974; DeVriend 1977). The zero order equation of motion describes the downstream stress field and integrates to equation (2). The first order equation groups the next smaller set of terms; in the case at hand this turns out to be the force balance for the cross stream shear stress field. It can be written as:

$$\frac{u_s^2}{r} = g\,\frac{\partial \eta}{\partial r} - \frac{1}{\rho}\,\frac{\partial \tau_{zr}}{\partial z} \tag{3}$$

where u_s is the downstream velocity at distance r from the center of the curvature for a segment of the bend, g is the gravitational acceleration, η is the free surface elevation above an appropriate reference level and τ_{zr} is the shear stress component in the cross stream direction on a plane parallel to the mean stream bed. The first term in equation (3) is the centrifugal force per unit mass, which is balanced by the cross stream pressure gradient due to the cross stream slope, the water surface, and the vertical divergence of the cross-stream shear stress. The last term represents internal friction caused in this case by the turbulent diffusion of momentum from the interior to the low momentum region near the bed. The stress divergence term can be moved to the left-hand side of equation (3) so that successive integration in the vertical and cross-stream directions, then division by g, yields an expression for the elevation difference (superelevation) across the channel. When equation (3) is derived in a cylindrical coordinate system after terms have been eliminated by symmetry (e.g., B. Yen 1972; Rozovskii 1957), r must be held constant with respect to downstream position. Nevertheless, authors such as Engelund (1974) have permitted r to vary according to a sine-generated curve, prompting Smith and McLean (manuscript in preparation) to derive an analogous expression in a curvilinear coordinate system appropriate for natural meanders. In the latter formulation r is a local radius of curvature and thus a function of downstream position, not a spatial coordinate. It should be noted that this more general approach is required if equations (2) and (3) are to be applied properly to the entire bend as described in this paper. The second order equation produced by the proper expansion is substantially different from that obtained using cylindrical coordinates.

FIELD SITE

Muddy Creek, a lowland tributary of the Upper Green River in western Wyoming, was chosen for the study largely because of previous and ongoing work by Leopold and Emmett (1976) on the East Fork River to which Muddy Creek is a principal tributary and by Andrews (1977) on Muddy Creek itself. At the measurement site the stream has a drainage area of 235 km^2 and a gradient of 0.0015. Bankfull discharge, mean width and mean depth are respectively 1.2 m^3/s, 5.5 m, and 0.6 m, and the maximum pool depth at this stage is 1.1 m.

During late spring and early summer the channel receives irrigation return flow and experiences long periods of relatively constant high discharge, which furnish the opportunity for detailed measurement of flow and sediment transport fields at a constant stage. A large proportion of the sediment in the channel originates from bank erosion in outwash terraces (Andrews 1977), so the supply of sediment is not exhausted early in the high-flow season. At high discharge the stream transports a coarse sand bedload in the form of well developed quasi two-dimensional sand waves in the curved reaches and complex three-dimensional dune fields in the crossings.

FIELD MEASUREMENTS AND DISCUSSION

Three sets of measurements were made over a narrow range of discharge. They included: (1) specification of velocity direction and magnitude throughout the water column to define the flow field and to calculate local boundary shear stress; (2) determination of water surface elevations along the banks of the meander to evaluate the magnitude of the superelevation and to guide computation of water surface topography; and (3) quantification of bedform geometry, local rates of bedform sediment transport, and average bed particle size. Measurements were made at the 10 cross sections shown in figure 1. Stage height was recorded throughout the investigation. All data were collected while the observer waded upstream in order not to disturb the bedforms at each measurement site.

Flow through the bend.—Magnitude and direction of the velocity were obtained separately. Vertical profiles of flow speed at 160 locations covering 10 sections across Muddy Creek were made with a cylindrical electromagnetic current meter. This instrument measures 6.25 cm by 3.75 cm and can be placed within 3 cm of the bed without causing scour. At the research site the current meter was attached to a top-set wading rod and oriented in the flow by maximizing the reading. An average velocity for each of five positions in the profile was estimated after watching an output dial for thirty seconds. Large scale turbulent fluctuations resulted in an error of approximately $\pm 10\%$ in the estimated values. The mean direction of flow at selected elevations was made visible with a short colored tape attached to the top-set wading rod, and its azimuth was read from a compass.

Our velocity data agree with the general flow pattern described by Leopold and Wolman (1960, p. 780–781). The major feature of this pattern (fig. 2) is a high velocity core that shifts from the inside to the outside of the channel with distance through the bend. Away from the core the velocity decreases rapidly. At station 10 the narrow core runs along the outside bank of the upstream bend; as it enters the straight section it continues over the deepest water against the same bank (near section 12). Upon entering the bend the flow experiences a rapidly

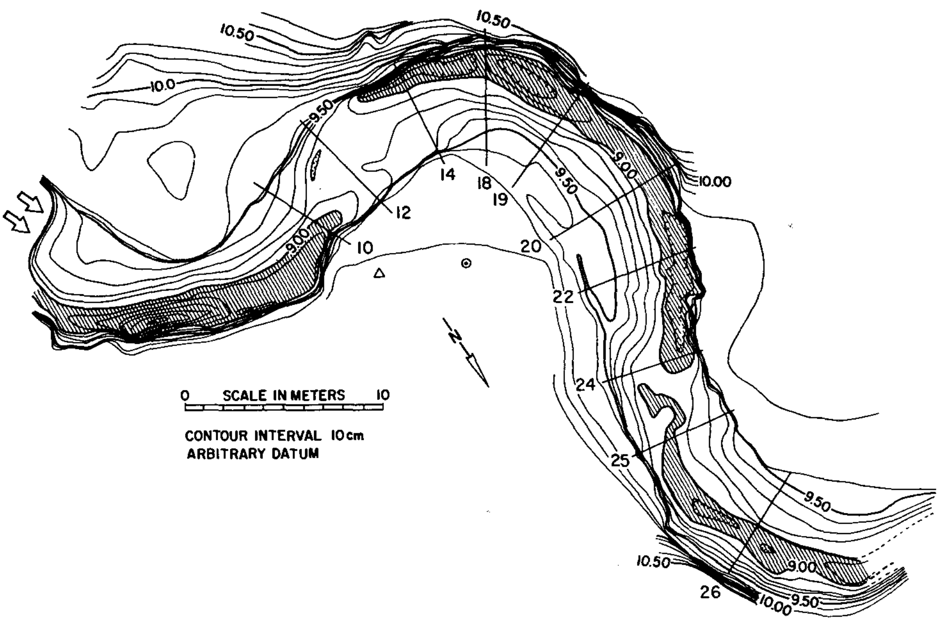

FIG. 1.—Topography of the Muddy Creek study site. The pools have been shaded for emphasis. The ratio of radius of curvature to bankfull width is 1.5 for the two upstream bends and 2.4 for the downstream bend. The center of curvature for the reach between sections 14 and 22 is indicated by the dot enclosed in a circle.

increasing centrifugal acceleration as the radius of curvature changes from infinity at the crossing to less than 10 m at section 14. In response, the water surface declines quickly along the inside of the bend and rises along the outside bank. In Muddy Creek and in most other well studied cases (e.g., Leopold et al. 1960; Ippen and Drinker 1962) the quick increase in superelevation after the cross over region produces an upstream slope along the outside part of the channel (from sections 14–10 and 26–24). Flow entering this region must decelerate because of the prevailing slope (as implied by equation (2) and as described by Ippen and Drinker 1962), and because of the rapid cross stream migration of the deep part of the channel downstream of the point bar (an inertial effect). C. Yen (1967) also recognized a deceleration resulting from horizontal flow separation caused by rapid widening of the deep water region at the downstream end of the point bar.

The superelevation created by the centrifugal acceleration on the high velocity near surface flow generates a cross stream pressure gradient which forces the slower fluid near the boundary to move towards the inner bank. Outward motion near the surface and inward flow near the boundary gives rise to the helical secondary circulation, discussed by Leopold and Wolman (1960) and others. From stations 14–20 the secondary circulation advects high momentum fluid outward over the pool, accelerating the flow there. Low momentum fluid is carried inward near the bed, decelerating the flow along the inside of the meander. In this manner the high velocity core remains intact but is shifted slowly toward the outside of the channel.

A simple calculation serves to illustrate the process of cross stream advection of momentum by the secondary circulation. The measured flow directions were used to compute the cross stream component of velocity for the region occupied by the 70 cm/sec isovel in figure 2. The cross stream velocities were 3, 7, 14, and 11 cm/sec between stations 12 and 14, 14 and 18, 18 and 20, 20 and 22 respectively. At section 19 the flow field was poorly defined so it was not used in this calculation. Multiplication of each cross stream velocity component by the time it takes the flow moving at 70 cm/sec to travel between

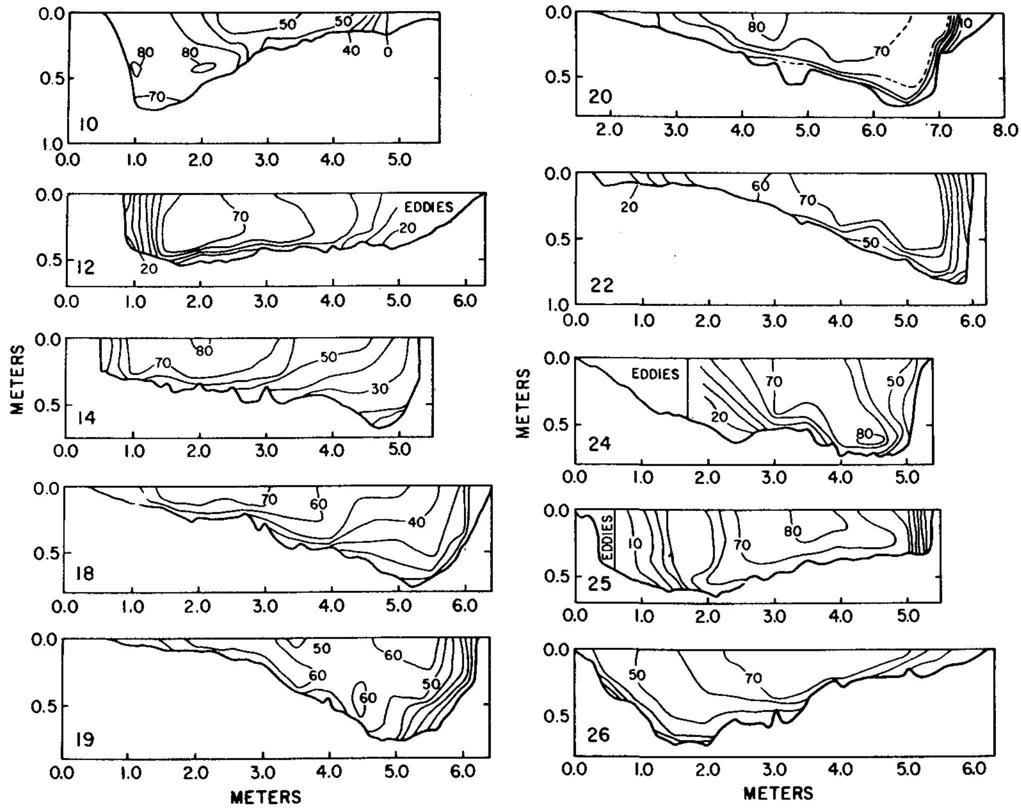

FIG. 2.—Upstream view of the cross sections of isovel surfaces for stations shown in figure 1. Measurements made at 0.7 times bankfull discharge between sections 10 and 19 and 0.8 times bankfull discharge between sections 20 and 26. Contour interval is 10 cm/sec. At each cross section the velocity field is defined by five measurements in a vertical profile at 0.5 m intervals across the channel. The large number in the lower corner is the station number.

sections gives an estimate of the rate of outward displacement of the high velocity core. The total displacements with respect to the initial location along station 12 were successively 0.22, 0.56, 2.25, and 2.80 m. Within the accuracy of the calculation, the values agree with the measured displacements of 0.35, 0.60, 1.6, and 3.3 m for the center of the high velocity core.

The measured velocity field in conjunction with the integrated form of equation (3) can be used to calculate superelevation in the bend. Further, the force caused by the vertical variation of the cross stream shear stress can be shown to be small in the integrated equation, so the superelevation can be computed just from measurements of the cross stream variation of vertically averaged downstream velocity along a specified radius. Predicted values, using vertically averaged velocities, were compared with measured cross stream differences in water surface elevations made at stakes at each cross section (except 19 where the stake did not stay in place). For sections 14, 18, 22, 25, and 26, measured values ranged from 9–22 mm and predicted values fell within 0, 11, 10, 19, and -14% of the respective measured values. The mean velocity near the outside bank at station 20 (fig. 2) was not sufficiently well defined to use this test.

Boundary shear stress distribution.—A standard method for computing local boundary shear stress at a point on the bed of a stream is to measure the vertical variation in flow speed near the boundary and then fit the results to the following equation:

$$u = \frac{u_*}{\kappa} \ln \frac{z}{z_0} \quad (4)$$

where u is the velocity at an elevation z above the bed, u_* is the shear velocity, equal to $(\tau_b/\rho)^{1/2}$, τ_b is the local boundary shear stress, κ is von Karman's constant, and z_0 is a roughness parameter. Muddy Creek carries only small amounts of suspended load (e.g., Andrews 1977), so a suspended sediment stratification correction of the type described by Smith and McLean (1977) is unnecessary. Further, there is no reason to expect von Karman's constant to vary so it was taken to be 0.4, a more precise value being unwarranted for the purposes of this study.

In order to compute the local boundary shear stress resulting from skin friction and from momentum extraction caused by bedload transport rather than that arising from form drag on dunes and sandwaves (Smith and McLean 1977), only the lowest measurements of velocity were fitted by equation (4). These values were obtained from 3, 6, 9, and occasionally 4.5 cm above the bed. Where bedforms were present we defined velocity profiles only over the crests in order to confine our estimates of boundary shear stress to the region of most uniform flow. These precautions could not entirely overcome the imprecision of the equipment and methodology. Often local compression of the thin zone to which equation (4) applies was sufficient to degrade the results. Therefore, some computed boundary shear stress values may be in error as much as $\pm 50\%$, and 31% of the profiles could not be used at all. The latter yielded unrealistically small values of z_0 (relative to those calculated by the Nikuradse method, Schlichting 1968, p. 583) because of large errors in the flow measurements.

In spite of these uncertainties, the results are found to be internally consistent and the general pattern of boundary shear stress is clearly defined (fig. 3*A*). This pattern is similar to the laboratory results of Hooke (1975) in that the zone of maximum boundary shear stress is near the inside bank in the upstream part of the bend and then crosses to the outside bank as it enters the central segment of the bend. Much more

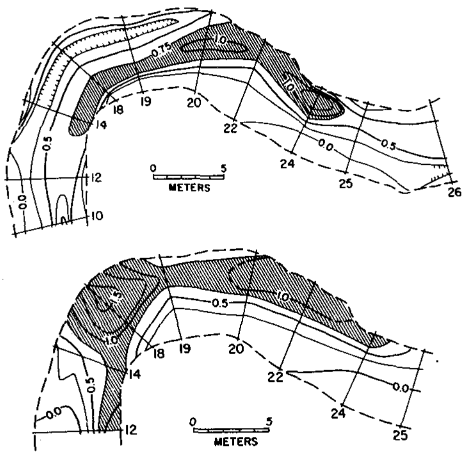

FIG. 3.—Boundary shear stress distribution. A: Measured distribution computed from velocity profiles. Discharge varied between 0.7 and 0.8 times bankfull during the measurements, and each computed stress is normalized by the mean total boundary shear stress in the downstream direction (calculated from equation (2)) between sections 12 and 24. The boundary shear stress is 50 dynes/cm^2 between stations 10 and 19 and 65 dynes/cm^2 between stations 20 and 26. B: Theoretical boundary shear stress distribution computed from equations (2) and (3). Magnitudes corrected to local boundary shear stress values by dividing by 2.11, and normalized by same values as in A.

pronounced than in Hooke's results are the long narrow regions of low shear stress which extend downstream from the lee side of each point bar far into the pool. In the lee of the point bar and in the upstream part of the bend along the outside bank, the development of a low boundary shear stress zone results from a very gentle water surface slope inclined upstream. Further into the bend as the flow crosses into the pool (fig. 4), inertia of the near bottom fluid carries the flow away from the steeply sloping boundary and reduces the vertical velocity gradient. The low boundary shear stress zone which results from this reduced velocity gradient persists until the flow is essentially along the pool rather than across it (section 20).

Equation (2) indicates that the boundary shear stress distribution can also be predicted from the downstream water surface slope and flow depth. Comparison of this prediction with the measured values in figure 3 illustrates the relative importance of frictional and inertial

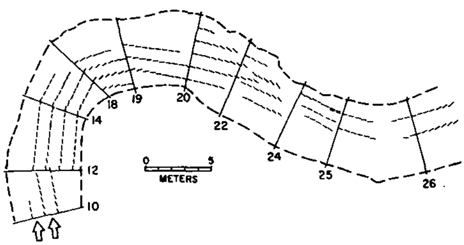

FIG. 4.—Direction of flow 2 cm above the crests of dunes and sandwaves. Discharges ranged from 0.70–1.26 of bankfull during the measurements.

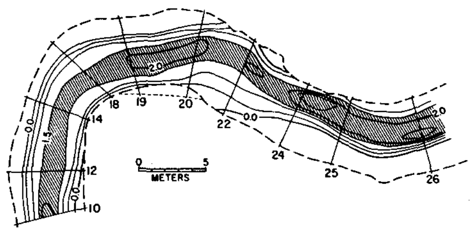

FIG. 5.—Bedload transport computed from downstream rates of bedform movement. The data are normalized by the mean bedload transport for the stage of measurement. Discharge ranged from 0.8–1.2 times bankfull and sediment transport ranged from 43–137 cm^3/sec (assuming a porosity of 0.3). Small dotted line between sections 18 and 22 is the local position of the right edge of water during stage of measurement.

forces in various parts of the channel. If the vertically integrated version of equation (3) is used to define the cross stream water surface profile between known values of elevation at each section, then the downstream slope from section to section can be computed from changes in water surface elevation along lines of equal radius. The average downstream slope and measured depth along successive radii across each section are then used to compute the boundary shear stress from equation (2). For each cross section the local slope at a given radius is defined as the average of the upstream and downstream slopes to adjacent cross sections. This averaging distance was of the order of 2–4 dune wavelengths and for that reason seems appropriate. In order to compare the total boundary shear stress computed by this procedure with the local boundary shear stress computed from velocity profiles, the predicted total boundary shear stress was divided by the average ratio of the average predicted shear stress to the average measured shear stress for each of sections 12 through 25 (fig. 3*B*). This ratio was 2.11 and can be attributed to form drag on the dunes and sand waves, to bank irregularities, and to form drag due to the bar-pool system.

Figure 3 indicates that equation (2) gives reasonable estimates of the gross pattern of boundary shear stress except in the central part of the bend (sections 18–19) where the inertial effects described above cause an enhanced near bottom momentum defect, hence a reduced boundary shear stress. Perhaps most importantly, these results indicate that, when the water surface topography is known, the zone of low boundary shear stress along the outside of the channel and the zone of high boundary shear stress near the inner bank at the entrance to the bend can be predicted by the frictionally dominated equations. Because of the large errors resulting from the method of data collection, a quantitative test of the cross stream distribution of boundary is not useful.

Bedform characteristics and sediment transport.—Four hundred and three measurements of bedform geometry and migration rates were used to estimate local bedload transport. Migration rates of individual dunes and sand waves are found to be remarkably constant for periods of up to two hours and over several wavelengths. Measurements of local bedload transport were expressed as a ratio to the average transport rate for each cross section and were plotted and contoured on a map (fig. 5). Because the measurements were made over a range of discharge (mean depth = 0.63–0.75 times bankfull depth) it is necessary to assume that the ratio of local transport rate to the mean value for a cross section did not change with stage over this range.

Comparison of figures 3*A* and 5 shows that the zone of maximum sediment transport is confined to the zone with a boundary shear stress ratio of 0.5 or more. The maximum boundary shear stress and maximum transport also correspond except for part of sections 10 and 14 and along the outside bank between stations 20 and 22. The lack of exact correspon-

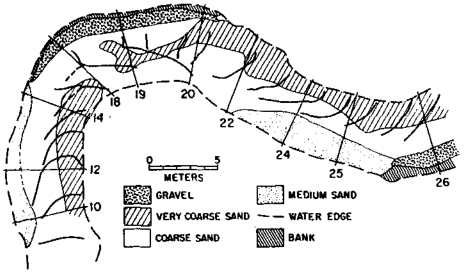

FIG. 6.—Texture of bed material at a range of discharge from 0.8–1.2 of bankfull and sketch of crest orientations of major bedforms at 0.7 of bankfull. The coarsest particles in the pool did not move at discharges of up to 1.3 times bankfull. In the zone between stations 14 and 19 the dashed lines indicate locations at which the slow moving outer extremities of crests merge as the high velocity of the inner crests increases the obliquity of the waves.

dence in these regions probably is due to a reduced ratio of bedform to bedload transport.

The zone of maximum particle size in transport (fig. 6) also corresponds to the core of maximum boundary shear stress. The zones of low shear stress and sediment transport, on the other hand, are occupied by the finest sediment at their upstream ends and the coarsest bed material (immobile at these discharges) at their downstream ends. Figure 6 also includes a sketch map of average spacing and orientation of the major bedforms. Short term or local distortions of the features that occur during downstream migration of the bedforms are ommitted from the map.

The bedforms are not locally perpendicular to the flow, but exhibit a variety of orientations while they migrate through the meander. As they emerge from the pool at cross section 10, the bedforms are convex downstream but have a mean crest direction perpendicular to the flow. The convexity decreases during the approach to the upper end of the point bar, and the mean crest direction becomes strongly skewed relative to the flow direction. In the upstream part of the bend, the right-bank (inner) end of the wave lies in the zone of highest shear stress and travels faster than the opposite extremity. Farther into the bend, the sand wave crests become straighter as the right-bank ends continue to travel faster and along a shorter path. These ends eventually decelerate and die out when the zone of high shear stress leaves the inner bank. Below section 19, the zone of highest boundary shear stress crosses to the outer edge causing the left-bank segments of the bedforms to accelerate and to bring the waves around more nearly perpendicular to the channel margins as they pass section 20. At this point the waves are slightly concave in the downstream direction. In the zone of highest shear stress, between cross-sections 20 and 25, the central sections of the bedforms travel fastest and the crests deform into a convex-downstream shape as they travel over a riffle. The form observed in the area of the upper riffle (sections 10–14) is repeated, therefore. Small ripples, oriented perpendicular to the flow, move slowly through the zones of low shear stress extending from the downstream face of the point bar into the upper part of the pool.

As suggested in the introduction, the bedforms that deform and migrate through the meander exert a strong influence on the flow directions near the bed and thus on the paths of particle movement. In the upstream part of the bend the flow on the stoss sides of dunes is parallel to the banks or slightly outward. According to our discussion of equation (3) the effects of the secondary circulation between sections 14 and 19 should cause a significant inward component of flow near the bed, yet we have observed in this reach a distinct outward flow over the crests (fig. 4). A solution to this apparent contradiction comes from the observation that along the obliquely oriented troughs of bedforms there exist currents toward the downstream part of the trough. These currents are sufficiently strong to transport large amounts of sediment, including granules, and are of great importance to sedimentation.

The trough-wise currents occur for two reasons. The case of a straight channel with obliquely oriented bedforms can be examined by means of the following analogy. In a long flume with upper and lower sections of uniform depth separated by a small negative step oriented in the cross flow direction, a vortex with negative circulation forms in the zone of separation leeward of the step. However, no cross stream velocity component is produced. If the step is made at an oblique angle to the flow then a vortex still develops on the lee side of the feature and its axis is still parallel to the topography. This means that the velocity just downstream of the step at the level of the upstream bed must be deflected perpendicular to the crest of the topographic feature producing a cross stream velocity component. In order to conserve mass, a cross stream pressure gradient must force an oppositely directed cross stream velocity component near the bed in the lee of the topographic feature and consequently a near-boundary flow occurs parallel to the axis of the vortex and in the direction of the downstream side of the flow obstruction. Material being transported as bed load in such a flow is carried toward the downstream end of the step. This process was investigated by Fredsoe (1974) and is illustrated in text by Raudkivi (1976, p. 70).

In a curving channel centrifugal acceleration of the whole stream produces a cross stream pressure gradient which exerts an inward force on flow near the boundary. If the obliquity of the bedforms is such that the velocity in the trough is low or inward, the cross stream pressure gradient will move this fluid toward the inside of the bend. However, over the crest the near boundary velocity is large and thus can have a centrifugal force equal to or greater than the force resulting from superelevation.

Observations of local directions and magnitudes of sediment transport in Muddy Creek indicate that the paths of particles moving near the boundary are dictated largely by the flow directions induced by bedform obliquity and secondary circulation. On the stoss sides of dunes and sand waves, particles entering the bend move toward the outer bank in the local direction of flow near the bed (fig. 4). After passing over the crest and tumbling down the lee side of the bedforms, these particles are caught by an inward current along the trough. Particles advancing inward along the trough are either buried by the avalanching crest face or are incorporated into the downstream and slightly outward flow on the stoss of the preceding dune. Each particle follows a zig-zag path, the average of which is not parallel to the channel banks.

By the time the sediment reaches the axis of the bend, the shear stress maximum has started to cross the channel, and the dunes begin to rotate into an orientation more perpendicular to the banks. Correspondingly, the intensity of the trough current and the magnitude of trough transport decrease substantially. In this section of the Muddy Creek bend, dunes extend across the sloping side of the point bar; as coarse grains avalanche from the crest they roll outward and down the trough (fig. 6). While in the trough, the coarse grains are subject to an inward trough current which is not competent to move them, and as they are alternately buried and re-excavated they follow a stepped path toward the pool. Finer particles that begin to avalanche down the crest face are sorted from the coarser sand and moved inward by the weak trough current. The gravitational force accelerating grains obliquely down the crest face toward the pool is proportional to the cube of the particle diameter: the fluid drag along the trough is proportional to the square of the particle diameter. The resultant of these two forces for small grains must therefore be further inward than that for larger grains.

After having crossed the channel the coarse sand follows the zone of maximum shear stress in the pool, along the outer bank, and onto the downstream point bar. Some of the fine sediment carried inward along the troughs is deposited in the zone of low shear stress downstream of the upper point bar. The remainder moves from point bar to point bar along obliquely oriented troughs of bedforms. These finer grains mix with the coarser particles as the paths of the size fractions converge and cross. Hence a strong sorting is produced between the pool and the point bar.

During a major stage change from low water (mean depth of flow of about 20 cm) to nearly bankfull (mean depth of about 50 cm) the pool in the central bend (fig. 1) scoured about 10–20 cm and the top of the point bar aggraded by a comparable amount. This scouring and deposition appears to be due largely to changes in the strength of the trough flow, caused both by increased superelevation and by changes in dune crest orientation. As the discharge increases there is a proportionately greater increase in boundary shear stress, and therefore in sandwave speed, over the point bar. Such a change in sandwave speed would increase the obliquity of the bedform crests. The result would be increased trough flow and sediment transport from the pool in excess of the material coming into this region, resulting in degradation. The sediment flux also would exceed the capacity of the flow over the bar to remove the sediment (causing aggradation). The ultimate depth of the pool is controlled in part by the reduction of the velocity gradient and therefore the shear stress as the boundary is lowered. At present our data are inadequate to test this model.

During periods of constant stage in which the cross sectional geometry is stable we have observed that the orientation, amplitude, and wavelength of the dunes and sand waves are roughly constant for any section of the bend. The relation between boundary shear stress field, dune crest orientation, and sediment transport along troughs suggests that an interactive adjustment leads to this stable geometry. For example, in the upstream part of the bend a small increase in obliquity of a dune crest would result in accelerated trough-wise transport of sediment to the downstream end of the dune. If when the cross stream sediment transport is added to the downstream transport at this point their sum exceeds the transport capacity of the local boundary shear stress, shoaling will result. This will force the high velocity core associated with the high boundary shear stress into deeper water. The outward shifting will cause both a deceleration of the crest speed at the downstream end of the dune and an acceleration in the deeper water. Resulting changes in orientation of the crest would decrease trough transport of sediment to the shoaled area. Without the contribution of sediment from the cross stream direction this area would erode and cause the high velocity flow to shift back inward, accelerating the crest speed once again. An equilibrium cross section may thus develop through a local balance of boundary shear stress and sediment transport which is achieved through adjustments in the bedforms.

CONCLUSION

A useful approach to understanding the physics of flow and sediment transport in a river bend is to consider the flow there to be frictionally dominated. Inertial forces may have important but secondary effects. The zero-order equation for frictionally dominated flow states that the boundary shear stress is proportional to the product of water depth and water surface slope. This equation gives a reasonable estimate of the gross pattern of boundary shear stress. It is incorrect, however, for the upstream part of the pool, where topographically induced inertial forces prevent the development of high boundary shear stress and therefore delay the crossing of the zone of maximum boundary shear stress to the downstream end of the pool. These inertial forces are important geomorphologically because they retard the scour of the pool and displace downstream the high shear stress which is partially responsible for bank erosion. Further documentation of the effects of inertia within the frictionally dominated flow model as a function of planimetric shape should lead to a quantitative theory for meander form.

In Muddy Creek large bedforms develop and respond to the boundary shear stress field by becoming oblique to the mean flow direction. Strong currents along troughs of sandwaves and dunes move sediment toward the point bar for the first two-thirds of the bend, preventing much sediment from entering the pool. At the downstream end of the bend the trough current transports sediment away from one point bar and toward the next one downstream. Variation in strength of the trough-wise current as the zone of maximum boundary shear stress shifts from the inside to the outside of the bend sorts the bed material. The interdependence of the sediment transport and shear stress fields through changes of bedform orientation suggests that equilibrium cross sectional geometry of a channel is attained through adjustment of the magnitude of trough-wise transport.

In most sand bedded rivers bedforms will develop and will dominate the local directions of bedload transport. In meandering rivers dune crest obliquity must occur because of the asymmetry of the boundary shear stress field. We suggest that close inspection of sand bedded rivers in which the bedforms and sediment transport at the bed are clearly visible will reveal that trough flow transport of sediment is common.

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