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Formation and controls of channel networks

by Thomas Dunne

I Statement of the problem

The geomorphological literature includes a vast number of papers describing the form of drainage basins (e.g. Strahler, 1964) and their responses to individual hydrologic events (Gregory and Walling, 1973), to climatic differences (Gregory and Gardiner, 1975), and to land-use changes (Strahler, 1958). Few authors have attempted to construct a theory of drainage network initiation and drainage basin development founded upon erosion mechanics. This paper reviews current knowledge about the physical principles governing the development of drainage basins through the initiation, growth, and integration of channel networks. The emphasis is on the relationship between these processes and the mechanisms by which runoff is generated, and on the degree to which current theoretical models of channel-network formation agree with field observations. The paper avoids discussion of the literature on random simulation of channel networks (e.g. Leopold and Langbein, 1962; Hack, 1965; Howard, 1971) and on the probabilistic-topologic approach to channel network description (e.g. Shreve, 1966; 1975). This literature, which was recently summarized by Jarvis (1977), is not concerned with the mechanics of erosion.

Drainage basins originate and spread in a variety of ways according to the history of the land surface on which they develop. Some channels develop almost simultaneously over the landscape and then integrate into the typical branched, hierarchical network. Typical examples are the development of gullies on highway embankments, artificial fills, cultivated fields, or on glacial till plains exposed by the downwasting of stagnant ice sheets. On a rising land surface channels extend downstream during slow warping or intermittent exposure of new land. The latter process was particularly obvious around Prince William Sound, Alaska, after the 1964 earthquake. The most commonly studied condition for channel initiation involves rejuvenation of the landscape by an increase in slope or by the lowering of a base level to generate a high gradient or a local flow convergence (Schumm, 1956; Morisawa, 1964; Abrahams, 1975). This last
case is the one most frequently studied by experiment (e.g. Parker in Schumm, 1977, 65–8), and research has focused on rates and patterns of channel growth rather than on the mechanics of incision.

In each of the cases referred to above, water falling or melting onto the unchannelled surface accumulates in various ways until there is a sufficient condition for the initiation of a channel. The resulting channels converge to form a branched hierarchical network that stabilizes relatively quickly in geological time, but which may undergo slow changes thereafter. The manner in which water accumulates to generate the conditions necessary for channel initiation involves the study of runoff processes, which has developed rapidly since Horton’s (1933) classic work (see Kirkby and Chorley, 1967; Freeze, 1974; 1978; and Dunne, 1978 for reviews of theoretical and field studies). Briefly, there are four paths that water can follow on its way downslope (see Figure 1). They are: Horton overland flow (path 1), groundwater flow (path 2), shallow subsurface flow (path 3), and saturation overland flow (path 4), and they are described in detail elsewhere (Dunne, 1978), along with the range of environmental conditions under which each process occurs. It will be argued later in this paper that each of these flow processes can contribute to the mechanisms by which channel networks and drainage basins develop.

II Channel initiation by Horton overland flow

1 Review of theory

As with many other important questions concerning the interrelationships between geomorphology and hydrology, the earliest attempt to explain the

![Figure 1 Potential paths of runoff. Path 1 is Horton overland flow; path 2 is groundwater flow; path 3 is shallow subsurface flow; path 4 is saturation overland flow composed of infiltrated water that emerges from the ground (return flow) and direct precipitation onto the resulting saturated zone. The unshaded zone indicates highly permeable topsoil, and the shaded zone represents less permeable subsoil or rock](http://ppg.sagepub.com)
formation of channel networks and drainage basins on the basis of runoff
mechanics was made by Robert Horton (1945, 331–69). His theory first
considers the mechanics of hillslope erosion and then the transition from
distributed erosion on hillsides to concentrated channel erosion. Where
Horton overland flow is the dominant runoff process, a thin, irregular
sheet of water flows down hillsides and imposes a shear stress on the soil
surface. The magnitude of the average total boundary shear stress (τ) at
any point on a hillslope is

$$\tau = \rho g d \cos \theta \sin \theta$$  \hspace{1cm} (1)

where ρ is the fluid density, g is the gravitational acceleration, d is the
mean flow depth measured vertically, and θ is the local angle of slope.
Most of this shear stress is exerted on vegetation, large immobile grains,
and tiny bedforms. An unknown fraction of the total boundary shear
stress is available for initiating erosion and transporting sediment, but for
the present purpose the discussion will be followed in terms of total
boundary shear stress, which varies along the hillslope profile with the
product d cos θ sin θ.

Horton showed that during steady-state turbulent runoff one can relate
shear stress to its more fundamental controls by using the Manning
equation to develop Equation 1 into

$$\tau = \rho g (n(i - f)x)^{0.6} \cos \theta \sin^{0.7} \theta$$  \hspace{1cm} (2)

where n indicates the Manning roughness coefficient, i is rainfall intensity,
f indicates the infiltration capacity, x is the horizontal distance from the
divide, and all units are in the mks system. Total boundary shear stress
increases with distance downslope for a constant or increasing gradient up
to about 40°. The equivalent expression for laminar flow is quite similar.
Horton (1945, 317) also pointed out that a soil has a resistivity, R, to
sheetwash erosion. Near the divide the eroding stress is less than this
shear resistance, but exceeds R at some critical distance, x_c. Setting the
value of τ in Equation 2 at this distance equal to R, Horton then solved
for x_c in turbulent flow as

$$x_c = \frac{R^{1.67}}{(\rho g \cos \theta)^{1.67} (i - f)^{1.17} \sin^{1.7} \theta}$$  \hspace{1cm} (3)

The critical zone over which sheetwash erosion does not occur (Horton
called it the ‘belt of no erosion’) is thus dependent on the shear resistance
of the soil, and varies inversely with the hydraulic roughness and slope of
the surface and with the runoff rate.

After discussing sheetwash erosion as a process distributed over large
areas of a hillside (pp. 319–326), Horton (1945, 331 ff) abruptly argued
that ‘Sheet erosion implies the formation of either a rilled or gullied
surface’ (p. 332), and he equated the critical distance, x_c, with the distance
required for the excavation of rills. He suggested that surface irregularities provide a local concentration of flow and of erosion which lowers the surface locally and diverts even more flow into the deepening rill, promoting extension of the rill upslope and downslope from the point of initial incision.

The implication that once sheetwash becomes erosive (i.e. for \( x > x_0 \)) it is inherently unstable and erodes small channels was taken up more formally in the valuable contributions of Smith (1971) and Smith and Bretherton (1972). The authors first define the equation of continuity relating sediment transport and the rate of change of elevation on a land surface, \( z = z(x, y, t) \):

\[
\nabla \cdot \vec{q}_s = -\frac{\partial z}{\partial t}
\]

(4)

where \( \vec{r} \) is the unit vector parallel to the local gradient, and \( q_s \) is the discharge of sediment per unit width of hillslope. Smith and Bretherton then postulate that \( q_s \) is governed by an equation of the form

\[
q_s = F(q, s)
\]

where \( q \) is the discharge of water per unit width of hillslope, \( s \) is the magnitude of the local gradient, and the function \( F \) is monotonic and positive for all \( q, s > 0 \). They later introduced a particular example of such a function, and I will use the particular form here for clarity:

\[
q_s = k q^N s^M
\]

(5)

where \( k, M, \) and \( N \) are empirical constants characterizing the specific sediment transport process (soil creep, rainsplash, or sheetwash). The one-dimensional form of Equations 4 and 5 was also used by Kirkby (1971) in developing hillslope profiles.

Smith and Bretherton used Equations 4 and 5 to compute one-dimesional 'constant form' hillslope profiles, which erode at a constant rate over the whole profile. Such a surface would be convex if \( M > 1 \) and \( 1 > N > 0 \) in Equation 5, but would be everywhere concave if \( M + 1 > N > 1 \). Kirkby (1971) pointed out that the former set of values is characteristic of soil creep and rainsplash erosion, while the latter set pertains to sediment transport by sheetwash.

The critical issue addressed in the Smith and Bretherton paper that is essential to drainage basin development is whether the hillslope profiles are stable against the tendency for sheetwash to erode channels. They formulated this problem by examining the change in local rates of erosion due to the introduction of one and two-dimensional perturbations in the hillslope surface (Figure 2). They found that the one-dimensional constant form surface is stable against small perturbations (Figure 2a), but that a two-dimensional constant form surface is stable against such perturbations (Figure 2b) only if it has a convex or straight longitudinal profile. On a
Figure 2  a) A constant form surface (solid lines) that has been perturbed only along the $x$-axis. The dashed lines represent the perturbations. For any sediment transport law $q_s = F(q,s)$ the constant form surface is stable.

b) The same constant form surface undergoing a perturbation in both $x$ and $y$ components.

Source: Smith and Bretherton (1972)

A concave surface, a small local convergence of sheetflow would grow into a channel.

A simplified analysis of the instability of a surface undergoing sheetwash erosion can be made by analysing the transport equation 5 under the assumption that $s$ is not altered to an important degree by a small two-dimensional perturbation. Such a perturbation causes a local increase in the water discharge, $q$, and since

$$\frac{\partial q}{\partial q} = Nkq^{N-1}s^M$$

(6)

sediment transport capacity will increase locally with water discharge if only if $N > 1$. The increased transport capacity will intensify erosion at the point of convergence, as indicated by Equation 4, and a channel will be incised. If $N = 1$ for a particular transport process (which according to the prediction of the original constant form surface would produce a straight profile), the local sediment transport capacity would not increase.
due to the flow convergence, and the surface would remain stable. Flow concentration would decrease sediment transport if $N < 1$, which would result in the filling of any small depressions. By the earlier analysis this last condition was associated with convex portions of hillsides.

Thus, Smith and Bretherton proposed that channels can develop only on concave segments of constant form hillslopes. Furthermore, they found that those disturbances with the smallest wavelength in the transverse ($y$) direction grow fastest. The latter statement is a puzzling and 'unrealistic' (Smith and Bretherton, 1972, 1521) feature of the prediction, the significance of which is not easy to apply to the field problem at this time.

The implication of this model, like that of Horton's theory, is that at distances greater than the critical $x_c$ defined by Horton sheetwash becomes erosive and develops a concave profile which is unstable with respect to small transverse perturbations. These latter grow into rills with intervening ridges, down which water flows into the deepening channel. If these ridges are shorter than $x_c$, they will be eroded by rainsplash and soil creep, but not by sheetwash and therefore will remain unrilled. These authors are thus in agreement with Horton about the significance of the critical $x_c$ and its controls, as summarized in Equation 3. The implication in both theories is that unchannelled sheetwash erosion does not occur in the landscape.

2 Evaluation

Contrary to the theoretical predictions, many concave hillslopes subject to sheetwash erosion are unchannelled. The author has conducted plot experiments on concave ungullied and unrilled hillslopes on the savanna of southern Kenya. During these experiments slight flow convergence and divergence were observed between and around the patchy grass cover (10–80 per cent), but no incision of a rill occurred. On bare soils, the sheet was particularly uniform across the plot, although one could see rapidly shifting, interlacing concentrations of bedload transport. In the same region, unchannelled concave hillslopes have lengths of several hundred metres. During natural rainstorms flow depths of several centimetres can be observed on the lower ends of these hillslopes, but no well-defined rills or gullies are formed in the upper few hundred metres of most hillslopes. These natural flow depths and those measured during the plot experiments exceeded the Shields criteria for sediment transport and were capable of transporting sediment without the synergistic effects of rainsplash. Therefore, it appears that the critical conditions necessary for the initiation of rills or gullies are not met simply where sheetwash becomes erosive, or even where it dominates sediment transport on a hillside. Channel heads are not developed at Horton's $x_c$, as he and Smith and Bretherton (1972, 1520) imply; another threshold must be attained for the onset of channelization.
This last claim is supported by data collected near Maralal in semi-arid northern Kenya, where I have measured erosion rates of up to several millimetres per year by mapping root exposures (see Dunne et al., 1978 for a description of the procedure). There is remarkably little evidence of rill or gully incision on these hillslopes, despite the intense erosion, long (>100 m) hillsides, and moderately intense rainstorms. Local concentrations of sheetwash occur, but the runoff diverges again after a short distance. Gullies develop on gradients exceeding 0.035 at hillslope lengths of 600–1800 m, and on gradients exceeding 0.10–0.15 at hillslope lengths 80–180 m. The variety of grazing land-use history has so far precluded a clearer definition of the critical combination of local gradient and drainage area (hillslope length) needed to trigger gully erosion. Patton and Schumm (1975) were able to define such a threshold combination for gully erosion on valley floors in Colorado.

The conclusion to be drawn from these observations is that in some rapidly eroding conditions, Horton’s critical length of sheetwash erosion is not the same as the critical length needed for the development of gullies. The latter distance is greater than the former by an amount that may be quite small in the case of some soils, but which in other locations such as Maralal and other parts of Kenya, is very large. The critical length for gully formation may be shortened in extreme rainstorms, and the gully may persist as a record of this event, or may even grow by erosion of a steep lip at its head.

3 Suggested modifications

The influence of rainsplash erosion may explain the incompleteness of theories on the instability of sheetwash erosion. If the flow over a slightly irregular surface is concentrated locally, the adjacent areas will be exposed to raindrop impacts either directly or through an attenuated sheet of water. Smith and Bretherton (1972, 1517) state that on the constant form surface (the results can also be generalized for a time-dependent surface, p. 1520) the downslope rate of perturbation of water discharge per unit width of contour \( \frac{\partial q'}{\partial x} \) is proportional to the local curvature of the perturbed surface elevation \( z' \) in the direction transverse to the flow. The transverse curvature \( \frac{\partial^2 z'}{\partial y^2} \) indicates the degree to which small irregularities on the surface force the flow to converge.

Thus

\[
\frac{\partial q'}{\partial x} = \frac{x}{s} \frac{\partial^2 z'}{\partial y^2}
\]

(7)

The implication here is that the greater the lateral curvature the greater will be the downslope rate of change of discharge and (from Equation 5) of sediment transport, and therefore the greater the rate of incision (from
Equation 4). However, the lateral curvature also governs the rate of lowering of a protuberance by splashing of soil particles laterally towards the depressions (or rills if the latter form). Thus if one takes the one-dimensional form of Equation 4 in the cross-slope direction

$$\hat{h} \frac{\partial q_t}{\partial y} = -\frac{\partial z}{\partial t}$$

where $\hat{h}$ is the unit vector parallel to the $y$-axis, and combines it with a commonly assumed transport law for rainsplash sediment transport of the form

$$q_t = k \frac{\partial z}{\partial y}$$

then

$$-\frac{\partial z}{\partial t} = k \frac{\partial^2 z}{\partial y^2}$$

The last equation indicates that rainsplash erosion should cause the lateral diffusion of sediment from protuberances to depressions in a manner such that the rate of lowering of the protuberance is proportional to the local transverse curvature, thus countering the aforementioned tendency for rilling of the depression.

Two other aspects of rainsplash erosion are also important for this tendency to counter rill incision. The first is that as a sheet of water thickens over a soil surface it drastically reduces the pressure and shear on the soil particles due to the impact of a raindrop (Wenzel and Wang, 1970). Mutchler (1967) has also shown that the range of a splash trajectory decreases as the sheetwash thickens, and experiments by C. J. McCarthy (personal communication) indicate that the volume ($V$) of sediment moved by a raindrop of a fixed size falling onto a flat surface varies with water thickness ($d$) according to

$$V = Ce^{-yd}$$

where $C$ and $\gamma$ are empirical constants for a raindrop size and sediment texture. It is difficult to isolate the effect of water thickness on sediment discharge by rainsplash alone because of the difficulty of conducting the necessary experiments with still water on a sloping surface. However, these results suggest that thickening of sheetwash should strongly decrease the efficacy of raindrops to move sediment laterally. The second aspect of rainsplash erosion that resists rill incision is its frequency relative to that of sheetwash erosion. Rainsplash exerts its levelling influence in many storms that do not generate Horton overland flow, and also acts alone during the early stages of many storms that do produce runoff.

Thus, beyond Horton’s critical $x$, one should expect sheetwash erosion,
but the tendency for the flow to concentrate might be offset by rainsplash erosion on the intervening protuberances. Sediment moving from these protuberances into the flow concentrations would partly satisfy the transporting capacity of the latter, thereby slowing their rate of incision. Equation 7 predicts that for a given gradient and transverse curvature the degree of discharge perturbation will increase downslope. The effect will be augmented as gradient decreases along the concave profile. The local rate of change of sediment transport capacity will increase accordingly as specified in Equation 5. A point \((X_c > x_c)\) will be reached at which the tendency for incision will exceed the countervailing process of levelling by raindrops. It is also possible that as the discharge increases downslope convergence of the sheetwash does not expose the protuberances to rainsplash until the incision is so deep that it cannot be erased by rainsplash during the storm or during later storms that do not generate overland flow. An implication of Equation 6 is that in more erodible soils (i.e. those with higher values of \(k\)) there is a greater tendency for channel formation for fixed values of \(N\), \(M\), \(q\), and \(s\). There also may be a requirement for some degree of cohesion in the soil in order to support a channel margin, but this is unproven.

### III Integration of channel networks formed by overland flow

In Horton's scheme of drainage basin development the first stage of channel initiation results in a system of parallel rills or gullies. The spacing of these channels constitutes another fundamental problem that has not yet been addressed by theories of drainage basin formation. Smith and Bretherton (1972) hypothesized that channel spacing was determined by the tendency for streams to migrate laterally so as to equalize the fluxes of sediment down opposite hillslopes, but this assumption leads to difficulties which the authors review. Much more theoretical and experimental work remains to be done on the processes governing stable channel spacings.

According to Horton, the parallel channels evolve into a dendritic drainage network through the processes of micropiracy and cross-grading. The bottom of the longest and deepest rill lies below that of adjacent rills, and caving or overtopping of the divide will divert water and sediment into the former, which therefore deepens further. The slope of the original surface is gradually replaced by small hillslopes the gradients of which have a component towards the master rill. If such a hillslope is longer than a critical distance (interpreted here as \(X_c\)), a tributary to the master rill develops on it. Horton (1945, 339-49) illustrated how repeated cross-grading and tributary development in this manner might produce a branched, hierarchical networks of channels, the density of which is eventually limited by the need for a critical hillslope length at which
channel incision can occur. As a drainage basin expands on a land surface emerging from the sea, new channels form as soon as the length of overland flow exceeds $X_c$. Streams which form first and have the largest drainage areas and lowest channels absorb their neighbours through micropiracy and cross-grading to produce basins with an ovoid plan form.

These processes of micropiracy and cross-grading can be observed in some small rill systems but it is inconceivable that overtopping or caving of divides could be the cause of channel integration in large valleys. Possible exceptions are drainage networks such as those on the Ontonagon Plain of Michigan, which according to Hack (1965) became integrated by random lateral channel migrations when ridges between streams were eroded by wave action during intermittent retreat of a glacial lake. Others (e.g. Howard, 1971) have proposed that drainage networks become integrated by stream capture, but at present the argument is based on the success of randomized computer simulation of the topologic characteristics of the network, rather than on field observations of evolving channel networks or on a consideration of erosion mechanics. Headward migration of channels in rejuvenated networks has been documented in the field (e.g. Schumm, 1956; Morisawa, 1964; Abrahams, 1975), but not studied mechanically. In general, there are major deficiencies in our understanding of the fundamental mechanics of channel network integration and therefore of basin form.

IV Horton controls of drainage density

The conditions under which channels are initiated by Horton overland flow are therefore still not precisely known, but occur at some critical channel-forming distance, $X_c > x_c$. The difference between these two lengths should decrease as rainsplash erosion declines relative to sheetwash erosion due to boundary shear stress alone, if the earlier discussion of smoothing by rainsplash is relevant. The two lengths should converge as the infiltration capacity decreases, as raindrop size decreases, as gradient increases (because the exponent in Equation 6 is greater for sheetwash erosion than for rainsplash), and possibly as the soil cohesion and resistance to dispersal increases.

However, since both critical lengths are partly controlled by the relative magnitudes of local boundary shear stress and the resistance of soil to erosion, $X_c$ should be subject to many of the same controlling factors that affect $x_c$. Horton (1945) was the first to summarize these controls in Equation 3, and he pointed out that the drainage density of a basin is approximately equal to one-half the reciprocal of $x_c$ (here interpreted as $X_c$). The drainage density, which determines many morphological and dynamic features of the basin can therefore be related to the climatic, edaphic, vegetative, and (in some cases) tectonic features of the landscape.
The analysis of variables controlling drainage density has been pursued by many geomorphologists. Melton (1958), for example, found by statistical analysis that in small subhumid catchments of the western United States, drainage density varied positively with a representative rainfall intensity, a representative runoff intensity, and with the percentage of bare soil. It was negatively correlated with infiltration capacity and a precipitation-effectiveness variable that is positively associated with vegetation density. The data did not show a significant correlation between slope and drainage density, although they were both weakly correlated with basin area in a way that suggested that gradient and drainage density both declined as basin size increased. Melton interpreted his findings in the light of the Horton theory, which they confirm.

Horton's theory was also confirmed qualitatively by field studies of Schumm (1956) on the development of small drainage basins in a vegetation-free, sand-clay fill. In particular, Schumm confirmed that the initiation of a channel requires the accumulation of runoff from a minimum area. He concluded that ‘... a sharp drop in frequency [of ungullied interbasin slope lengths] at 10 feet suggests that at lengths above this runoff surfaces are unstable in form and will tend to develop channels’ (Schumm, 1956, 611). Small rills, which developed during each rainy season and were obliterated by frost action each winter, fed into these channels and extended to within 1.5 feet from the divide on 48° slopes. It was not, however, possible to study the mechanics of incision. Abrahams (1975) made similar interpretations concerning the headward growth of small channels in a valley fill. He found that there were fewer short cis links (adjacent tributaries entering from the same side) in the network than short trans links (adjacent tributaries entering from opposite sides). Abrahams pointed out that during headward growth the formation of a new tributary on the same side as a down-stream tributary is deferred until the main channel has extended some critical distance beyond the last tributary. In this way the minimum contributing area is provided for tributary formation on the same side of the main channel. Tributaries forming on opposite sides of the channel do not compete for drainage area.

V Channel initiation by subsurface flow

Until the mid-1960s the Horton model of channel initiation and drainage basin development was accepted for both humid and subhumid environments. Thus, Strahler (1958), Chorley (1957), Chorley and Morgan (1962), and many others reasonably interpreted the morphometry of humid landscapes in the light of the controlling variables proposed by Horton. In particular, temporal and interregional contrasts in drainage
density were related to differences in precipitation and runoff intensities, basin steepness, and vegetation or an index of plant production. The explanations presented for the observed relationships were those of Horton, although the only direct field evidence for the Horton runoff mechanism in humid regions appears to have been from small devegetated basins (Strahler, 1958). It is now generally accepted that subsurface flow and related saturation overland flow (Figure 1) convey runoff to channels in humid regions; the development of this idea is reviewed by Chorley (1978). Kirkby and Chorley (1967, 8) pointed out that subsurface flow and its influence on erosion processes provide 'the other end-member of the continuous spectrum of possible flow and erosion models'. They were referring to shallow subsurface flow (path 3 in Figure 1), but as will be described later, deep groundwater flow (path 2 in Figure 1) is also important in the development of channel networks and drainage basins.

1 Field observations

Dunne (1969) studied runoff processes resulting from rainfall and snow-melt in a small catchment in Vermont (Figure 3). The 3.9-hectare grass-covered basin had been forested until a few decades earlier. It was developed in a sandy and silty glaciolacustrine terrace lying against a major valley side composed of siliceous and calcareous granulite (Dunne and Black, 1970b, Figure 1). A railroad along the western edge of the catchment intercepted shallow sub-surface flow but not deep groundwater flow which originated on the major valley wall west of the margin shown in Figure 3. Groundwater moves to the stream through widely spaced major vertical joints in the bedrock. In the surficial deposits, shallow subsurface flow and saturation overland flow were documented during natural and artificial rainstorms (see Dunne and Black, 1970a; 1970b; 1971; and Dunne, 1978 for summaries of the results).

I then addressed the question of how branched, hierarchical channel networks could form in a humid region where the mechanism of channel incision proposed by Horton did not exist. It was also important to explain why the morphometric features of humid landscapes, particularly drainage density, could be related to the same controls that Horton, Strahler, and Melton had proposed, namely rainfall intensity (Chorley, 1957; Chorley and Morgan, 1962; Cotton, 1964), infiltration capacity (Chorley, 1959), the permeability and strength properties of the underlying rock (Carlston, 1963; 1966; Strahler, 1964, 4–52), relief (Strahler, 1958), and time (Ruhe, 1952).

In Vermont, geomorphological processes are most active during the snowmelt season, although some sediment may be moved from hillsides during rare summer storms and autumn hurricanes. During the snowmelt season 10–50 cm of water are stored in soils and rocks within a period of 10–30 days, and pore-water pressures usually reach annual maximum
Figure 3 Hydrologic and structural features of a small drainage basin in Vermont. The shaded zone indicates the area over which saturation overland flow occurred at some time. Structural features and labelled sites are described in the text.

levels during or soon after this time. During several years of observation in the small study basin, the following geomorphologically significant features were observed.

Overland flow occurred beneath the snowpack over large areas of the basin because of the presence of concrete frost, but the frozen soil was immobile and no erosion occurred. Saturation overland flow extended over the shaded zone in Figure 3 during autumn or spring, but since these areas generally had a thick vegetation cover and low gradient, surface erosion was virtually non-existent, except at a few seepage steps, similar to but smaller than those described by Hadley and Rolfe (1955). These seepage steps occurred most frequently around the lower parts of hillslope hollows and around the heads of some channels that developed on gently sloping terrain where the transition between channel and hillslope was gradual.
Subsurface hydraulic gradients became positive downward as water returned to the soil surface in some places. This occurred in both the deep groundwater system of the bedrock and in the shallow phreatic zones in the overlying sediments and soils. At some predictable locations the hydraulic gradients were large enough to cause piping of the soil (see later). At one of those locations (Site A in Figure 3) soil falls were observed around a spring head during the melt season.

The dip of bedding and the alignment of joints in the bedrock control the pattern of the topography and runoff-producing zones in the experimental basin. The bedrock dips eastward at 35–40° and near-vertical joints strike approximately east-west. The springs at A, C, and D in Figure 3 emerge from high-angle bedrock joints, the surfaces of which are friable and stained brown. A high pore pressure that developed at site A during the snowmelt season persisted for several weeks. Water from the springs maintains a remarkably constant temperature throughout the year and the discharge exhibits only slow seasonal fluctuations. These characteristics suggest that the springs are fed by deep groundwater that moves in the bedrock along bedding planes and major joints. The springs appear to have sapped headward along these major joints, to form the channel network.

Not all of the joints or intensely fractured zones have been invaded by a stream channel. At the upper (northwest) end of the valley several shallow linear depressions trend east-west on a hillside underlain by bedrock with a thin colluvial cover. The depressions are visible only at low sun angle and in the spring when they are sites of earliest grass growth. They contain brown-stained sandy soil with gravel-size fragments of the granulite and are separated by indistinct convex ridges of bedrock. During the snowmelt season the depressions are wetter than most hillsides in the basin and the zone of saturation overland flow extends upslope into their lower ends (Figure 3). These features are interpreted as seepage lines where the emergence of groundwater from high-angle joints causes accelerated chemical weathering, and eluviation, but where the critical hydraulic gradients necessary for piping and stream channel formation do not occur. This differential exploitation of the bedrock along the outcrops of major joints forms seepage lines reminiscent of those described by Bunting (1961) on the gritstone moors of Derbyshire.

In northern Vermont and areas of similar hydrology, channels appear to be initiated and valleys extended by spring sapping, a term that includes several processes involving the role of subsurface flow as it returns to the ground surface. The initial step in spring sapping involves mechanical weakening of the rock by chemical weathering and frost action, which lower the cohesion and tensile strength of the residuum, while its porosity and hydraulic conductivity are usually increased. As water is forced through the residuum by a pore-pressure gradient it exerts a drag on the weakened material. This gradient varies spatially and
temporally, as will be discussed later, and at certain critical times and places reaches a level that overcomes the declining resistance of the weathered bedrock to failure by piping.

2 Theoretical considerations of spring sapping

Hydraulic piping has been defined by Källin (1977, 107) as 'a type of mechanical, intraformational erosion caused by the flow of groundwater'. Terzaghi (1943) had defined the critical pore-pressure conditions required for the simplest case of the instantaneous piping of a relatively large mass of non-cohesive material on a horizontal surface. The magnitude of the vertical component of the pore-pressure gradient at the surface must be large enough for the upward drag force on a particle or aggregate at the surface of a porous medium to exceed the immersed weight of the grains. Thus, it is necessary that

$$-\rho g \nabla \phi \cdot \hat{n} > (\rho_i - \rho_f) g (1 - \rho)$$

where $\rho$ is the density of the (assumed single) mineral phase, $\rho_i$ is the porosity of the weathered residuum, $\nabla \phi$ is the gradient of hydraulic head in the fluid near the surface and $\hat{n}$ is the unit vector normal to the surface. The associated velocity of flow must exceed the settling velocity of the grains or aggregates in order to carry them away.

Zaslavsky and Kassiff (1969) extended the analysis of piping to include the case of cohesive materials on a sloping surface (Figure 4). Three forces act on a particle or aggregate at such a surface, and for piping failure to occur their resultant must have a component normal to the surface and directed outwards; i.e. in the direction of the normal unit vector $\hat{n}$) in Figure 4. The normal component of the weight of the volume ($V$) under consideration is

$$F_g = -V(\rho_i - \rho_f) (1 - \rho) g \cos \theta$$

(9)
where the negative sign indicates that the force acts downward and therefore stabilizes the slope. The fluid drag on an individual particle or volume of soil was assumed proportional to the macroscopic drag force averaged over many particles, and thus can be written as

\[ F_d = -c_1 V \rho_f g \nabla \Phi \cdot \hat{n} \]  

(10)

where \( c_1 \) is the proportionality and is related to the shape of the soil element. By application of Darcy's Law, one obtains

\[ F_d = c_1 V \rho_f g Q \cdot \hat{n} \frac{Q}{K} \]  

(11)

where \( Q \) is the specific flux vector (discharge per unit area) of water returning to the ground surface, and \( K \) is the saturated hydraulic conductivity of the porous medium. The last parameter refers only to the immediate vicinity of the surface, and is not easily measured in the field, although for the present purpose it seems reasonable to assume that it bears some simple relationship to the hydraulic conductivity of the bulk of the underlying soil or weathered rock.

The vectorial sum of \( F_g \) and \( F_d \) is resisted by a cohesive force of opposite sign and a maximum magnitude of

\[ F_c = c_2 D^2 \sigma_t \]  

(12)

where \( c_2 \) is another geometric coefficient relating the particle diameter \( (D) \) to the projected area of particles normal to the resultant of the driving forces, and \( \sigma_t \) is the tensile strength of the soil which can be obtained by projecting the strength envelope on a Mohr diagram to the normal-stress axis and reading the negative intercept at \( \tau = 0 \). At piping failure

\[ F_c = F_g + F_d \]  

(13)

and for cohesionless soils \( (F_c = 0) \) on a flat surface this expression reduces to Equation 8. The foregoing analyses assume that no piping erosion takes place until the critical hydraulic gradient develops. Kälin (1977) pointed out that piping is often a stochastic process in which the sequential removal of a relatively few susceptible grains can lead to a gradual loosening and eventual reworking of the whole mass.

Measurements of hydraulic head in water returning to the ground surface in the catchment shown in Figure 3 allowed the evaluation of piping conditions at a few sites. At site A, a piezometer was placed in a bedrock joint from which water was welling upward toward the horizontal ground surface. The bedrock surface lay 60 cm beneath a cover of cohesionless soil and till with a porosity of 44–54 per cent immediately downslope from an 80 cm-high scarp of the same material. For several weeks after the snowmelt, the water level in the piezometer stood 60 cm above the ground surface, indicating a vertical hydraulic gradient in Equation 8 sufficiently strong to cause piping immediately above the
bedrock joint. The washing away of piped material from that region undermined the spring-head scarp which retreated 30 cm. A similar occurrence during the following snowmelt season caused a 10 cm retreat. Piezometric measurements were also made in water returning to the ground surface at sites B, E, and F (Figure 3) in the Quaternary sediments of the basin. During the snowmelt season only weak hydraulic gradients built up at site B, but during a simulated 50-year, one-hour rainstorm the vertical component of the hydraulic gradient was at least 0.5, whereas soil piping there would require this value to be 0.83. No piping occurred at the site, but a small quantity of soil was detached from the head of the small channel two metres downslope of the piezometer. At site E on a steep, permeable hillside and site F in the marshy valley bottom hydraulic gradients reflected the emergence of subsurface flow during the snowmelt and during large rainstorms, but did not approach the critical values necessary for piping.

VI Drainage network development by spring sapping

Small and Lewin (1965, 8-12) suggested that valleys develop mainly by spring sapping in the Chalk escarpment of southern England. Unaware of this work, I formulated a conceptual model of channel network development by spring sapping based on observations described above. I proposed that this is the dominant mechanism generating drainage networks and basins in many humid regions on a variety of rocks (Dunne, 1969). Some of the features of the model have been put forward by other writers, particularly for small, shallow drainage channels in unconsolidated materials (e.g. Jones, 1971; Löffler, 1974; Baillie, 1975). De Vries (1976) has also described a conceptual model of drainage basin development as a result of groundwater flow in the Holocene landscape of the Netherlands, although the erosion mechanism was not specified.

In the following description I will consider the simplest case of drainage-pattern evolution after a single rapid tilt that brings a smooth land surface of permeable rock above sea level. It is possible to envisage more complicated antecedent histories, including the deglaciation of a randomly irregular topography, without altering the major features of the proposal. The effect of extremely low rock permeability will be discussed later.

Elevation of the rock mass above base level causes a slope of the regional water table and the drainage of groundwater towards the hydraulic sink provided by a stream or sea level (Figure 5a). The flow and potential fields in such a system can be calculated according to the methods introduced by Toth (1962; 1963) and extended by Freeze and Witherspoon (1966; 1967; 1968) for numerical integration of Hubbert's (1940) groundwater flow equations subject to a variety of topographic and
Figure 5  Plan view of a groundwater flow net during extension of spring heads to form a drainage network. Solid arrows are flow lines, dashes indicate equipotential lines. a) groundwater flows towards the land margin; b) convergence of groundwater flow at the head of an embayment produced by a small piping failure or by an initial irregularity in the land margin; c) increased convergence of flow lines around a spring head that has retreated headward from the land margin extending a valley. A second piping failure has occurred on one side of the valley and is distorting the flow field in that region. After Dunne (1969)

stratigraphic boundary conditions. Toth (1966) also demonstrated techniques for field mapping and testing of the predictions, which for the simple case being considered are shown in Figure 6. Beneath the upper part of the land surface there is a region of downward flow, and under the region near the outlet flow has an upward component of varying magnitude.

Chemical weathering by the emerging water increases the porosity of the rock and reduces its tensile strength thereby rendering the weathered material more susceptible to piping forces (Equations 9 and 12) during times of greatest hydraulic gradients. An associated increase in hydraulic conductivity in the weathered zone would tend to reduce the piping force,
According to Equation 11, but we shall see later that this effect is at least partly offset by the increased flow of water toward a piping location.

Even within a so-called uniform rock there are various degrees of heterogeneity in several properties. The two properties of greatest significance for the present discussion are the hydraulic conductivity and resistance to chemical weathering; they are also linked. Minor differences in grain size, porosity or jointing could give rise to a spatial variation of conductivity with wavelengths that vary from millimetres to kilometres or more. An example of the latter scale is presented by Ineson (1962) who mapped transmissibility in the English Chalk which is most permeable in zones of fine fractures and small faults along the axes of gentle anticlines. When such variations of conductivity do not occur initially, they may develop if there are spatial differences in mineral stability, which may also exist on a range of scales down to that of single grains. Differential weathering of the minerals by the groundwater would usually increase the conductivity in that small region.

Whether due initially to heterogeneous mechanical properties of the porous medium, or secondarily to differential chemical weathering, spatial variation of hydraulic conductivity may be expected to occur within the bedrock. Such variations should occur on a variety of scales and cause perturbations of the simple flow fields depicted in Figures 5a and 6. Flow would be concentrated towards the more permeable zones, which would therefore be subject to the most intense chemical weathering and would tend to become even more permeable than the surrounding rock. The degree of flow concentration would be proportional to the rate of change of hydraulic conductivity transverse to the flow line, and would be particularly strong in the vicinity of permeable joints or fracture zones. It is to be expected that some feature (joint spacing or mineralogic zonation) with a characteristic wavelength would grow in this manner faster than all other wavelengths, determining a stable characteristic spacing for the flow concentrations. However, it is not yet obvious what field measurement should be made on the bedrock in order to predict this proposed characteristic spacing.

The upward component of groundwater flow and the progressive, non-uniform decreases in density and tensile strength and increases in hydraulic conductivity and flow concentration over geologic time eventually generate the critical conditions for piping erosion at some point (Figure 5b). If sufficient heterogeneity already exists when the land surface rises above base level, piping occurs almost immediately. Initial irregularities of the land margin would also lead to flow concentration. Piping could involve the sudden removal of large masses of weathered rock, or the progressive removal of single grains or small aggregates, as considered by Kålin (1977).

The process excavates a spring head and causes local lowering of the ground surface. This change increases the probability of repeated piping
Figure 6 Calculated two-dimensional potential distributions and flow patterns for groundwater in aquifers of different thickness. Source: Toth (1962)
because groundwater flow lines concentrate even more intensely on the spring head. Increased flow accelerates chemical weathering and also increases the hydraulic gradient at the edge of the initial indentation (see Figure 5b and Equations 10 and 11). Thus a positive feedback mechanism exists whereby the initial failure increases the probability of future piping at the same site. The further a spring head retreats, the greater is the flow convergence, and the greater is the tendency for future spring sapping, so that the rate of headward erosion increases. Headward sapping proceeds faster than valley widening because the valley head is the site of greatest flow convergence (Figure 5c), although widening may take place as a result of lateral corrosion by the stream as well as spring sapping.

As the stream extends headward, it disrupts the simple two-dimensional flow pattern, and water emerging along the valley sides eventually exploits some susceptible zone (Figure 5c) to form a tributary which also undergoes headward retreat and branching. This type of development fits the prediction of Dacey and Krumbein (1976, 154) whose topological investigation led them to propose that the growth of stream channel networks occurs mainly by tributary development along one side or the other of a growing first-order channel, rather than through bifurcation at the growing tips of the channels. However, the spring-sapping hypothesis does not preclude branching at tips, or even the more or less simultaneous piping of unconnected spring heads and later integration of the resulting channels.

The process of repeated failure and headward retreat and branching forms a network of river valleys (Figure 7). Integration of the channel net-work

![Figure 7](image-url)
in this way is not limited to very small drainage basins as was the case with the overtopping of divides in Horton's suggestions about micropiracy and cross-grading. Subsurface water can migrate beneath topographic divides and the competition for groundwater leads to stream capture or differential lowering rates between large streams.

If the flow and the distribution of weaker zones are controlled by rock structure, various degrees of preferred orientation develop in the drainage pattern, ranging from the obvious, rectangular trellised networks of some regions to the more common dendritic patterns. The influence of structure may often be so subtle that it can only be recognized through systematic field and map measurements. For example, Newell (1970) conducted a series of joint surveys in the granulite of the Vermont study region. He plotted poles to joint surfaces on an equal-area graticule and contoured the results to define between one and three high-angle joint sets. He then plotted hillslope gradient vectors in the same manner and found a rough parallelism between the preferred orientation of the stream valleys and that of the fracture pattern. His interpretation was similar to the one under discussion.

Once the two-dimensional groundwater flow field has been disrupted, it is difficult to predict the consequences quantitatively by the methods of Toth, Freeze and Witherspoon, because the changing topography progressively distorts the flow field which in turn alters the topography. But the pattern can be discussed qualitatively and illustrated as in Figure 7. Headward migration or the piping of an increasing number of spring heads eventually counteracts the positive feedback mechanism of flow concentration referred to earlier. The development of neighbouring spring heads eventually decreases the drainage area of each, and limits the supply of water. In effect each growing valley competes for groundwater discharge in order to continue headward growth. The frequency of hydrologic events large enough to remove weathered material will decrease as the drainage of each spring declines. One might expect that a balance will eventually be struck in which the rotten rock produced by chemical weathering is permeable enough to pass the peak groundwater discharge from the reduced drainage area without the build-up of critical hydraulic pressure gradients. Colluvium, brought in by soil creep, may choke the valley head and be scoured out by piping on rare occasions, but the general location of the main valley head should be determined by seepage within the bedrock in areas where the valley form is expressed in the bedrock, as well as in the colluvium.

In humid mountainous terrain the valley head is often marked not simply by a spring but by a landslide scar or the initial failure of a debris flow, and many of these show signs of concentrated water flow from joints or funnel-shaped hollows in the bedrock where hillslopes are so steep that high pore pressures mobilize a failure without piping. Dietrich and Dunne (1978) described the process of failure and re-filling of such bedrock
hollows in the Coast Range of Oregon. Hack and Goodlett (1960) studied the results of valley-head scouring by a large storm in the Appalachians. Coaz (1869) provided a particularly vivid account of such failures during a violent rainstorm in the Alps. He reports (p. 11) that 'Most earth slips observed originated in seepy or boggy sites where both spring and atmospheric water was concentrated on more or less concave, impervious soil or rock layers'. Later (p. 74) he concludes that the slips were in part caused by concentrated surface runoff, but were mainly the result of internal saturation and 'eruption of excessive seepage'. His report reads like an extreme case of the runoff processes observed in the shaded area of Figure 3.

The bedrock in which groundwater sapping can develop drainage networks does not have to be an aquifer in the usual sense associated with water supply. The granulite studied in Vermont is not considered as an aquifer. Relatively small fluxes of groundwater could account for the processes described above over long periods of geologic time. However, it is likely that some rocks are so impermeable that sapping can occur only in shallow groundwater systems in colluvium. In such rock, channels are probably incised by chemical weathering, corrasion, and scouring by debris flows as water escapes from the terrain by spilling through random irregularities in the original terrain. The geologically youthful drainage system of parts of the Canadian Shield has probably evolved in this manner. A range of intermediate depths of groundwater penetration exists between this impermeable extreme and deep aquifers. Lawson (1968, Figure 7) has documented the flow field in one intermediate case.

**VII Subsurface controls of drainage density**

Under the groundwater sapping hypothesis, as in the Horton model, some minimum catchment area is required to provide sufficient water to generate and maintain a stream channel. The minimum area required to sustain a unit length of channel was called by Schumm (1956) the 'constant of channel maintenance', and is the inverse of the drainage density. It is now possible to examine the factors that should control this important morphometric characteristic of landscapes affected by the groundwater sapping mechanism.

The most obvious control involves the amount of water available for spring sapping, but the details of this effect are difficult to specify until it becomes feasible to construct a deterministic mathematical model of the effect of three-dimensional, unsteady groundwater flow. The most important measure of water availability might be the total annual runoff, which would affect the rate of chemical weathering along each groundwater flow line, including near the outlet. Probably of greater importance is some measure of short-term concentration of the water
input, which would control the build-up of the highest pore-pressure gradients. Thus, total winter precipitation or spring snowmelt might be the critical variable in many humid temperate regions. It seems likely that the deeper and larger the groundwater flow system determining the stream network, the more sustained would have to be the water input to generate piping conditions because of the larger capacity for storage and damping of the input. Shallower and smaller flow systems would develop critical pore pressures in shorter, more intense rainstorms or snowmelt. Thus, in Vermont, the bedrock flow system generated maximum pore pressures during the snowmelt and maintained them for several weeks; in the deep, permeable soil at site E (Figure 3) short-lived maxima occurred in response to daily snowmelt cycles and large, artificial rainstorms, while in the shallow soil at site B piping conditions were approached only during a simulated 50-year rainstorm.

It is difficult to isolate which hydrologic input is the dominant control of channel formation and drainage density by statistical analysis because of the high degree of correlation among the various rainfall parameters mentioned. The dominant control will probably differ between climatic regions and rock types, and can only be isolated through field measurement and deterministic modelling. It is only possible at present to say that the generation of groundwater flow and hydraulic gradients from an area varies directly with some measure of the precipitation, and that as a corollary the area required to provide enough flow to generate the critical hydraulic gradient required for piping varies inversely with precipitation. Thus, drainage density should increase with some measure of precipitation as discussed by Chorley (1957), Chorley and Morgan (1962), Cotton (1964), and De Vries (1976).

The intensity and form of chemical weathering and its effects on rock properties must also affect channel formation and drainage density. The intensity of chemical weathering varies with the amount of available water, carbon dioxide, and other acids, all of which are strongly controlled by climate. Lithology, the other dominant control of susceptibility to weathering has been documented elsewhere.

Various geological characteristics are important to the spring sapping mechanism. The first is the hydraulic conductivity in both the bulk of the formation and the immediate vicinity of the valley head. For a fixed rate of groundwater recharge per unit area of catchment, the hydraulic gradient around a spring head intensifies as the conductivity declines in that region, increasing the $F_d$ term in Equation 11 and increasing the drainage density. However, if the conductivity in the bulk of the formation is low, the fluid potential at the base of the weathered residuum (and therefore the hydraulic gradient within it) will be relatively low for a fixed recharge rate and relief, and the drainage density should be relatively low. Some of the correlations between infiltration capacity and drainage density have involved humid areas as well as basins subject to Horton overland
flow. However, it seems likely that any such correlation within humid regions reflects the influence of permeability on pore-pressure gradients and piping mechanisms in the weathered residuum.

Finally, relief should be an important control, particularly in shallow groundwater flow systems in weathered bedrock, soil, and surficial deposits, where the gradient of elevation head is the dominant component of the hydraulic gradient. Therefore, one might expect the positive correlation between drainage density and relief that has been proposed for regions subject to Horton overland flow to apply to humid regions, but for a different reason. De Vries (1976) referred to the influence of relief on stream density in the Netherlands.

Differences in vegetation density are not likely to have a recognizable influence on drainage density in most humid regions, so long as the infiltration capacity is great enough to preclude Horton runoff. Vegetation exerts competing influences on available water, chemical weathering, and the tensile strength of soils, and the net effect would be small and recognizable only in a few cases.

VIII Research needs

This review of current knowledge of processes that generate a branched, hierarchical channel network highlights several major unsolved problems in an important area of geomorphological theory. Resolution of these issues requires field measurements of uncommon thoroughness and detail as well as certain theoretical and computational developments. The field measurements should ensure that the theoretical analyses take into account the relationship between runoff mechanisms and the physics of the erosion processes that generate channels.

Under Horton overland flow much uncertainty remains about the incision of channels. It plausible that rill formation is a consequence of the form of the sediment transport law describing sheetwash erosion. However, there is need for a close examination of the physics of the initial incision, and of the interaction between rainsplash and sheetwash erosion in the zone upslope of $X$. Very little is known about what governs the stable spacing of the resultant channels or of how they become integrated into networks draining pear-shaped basins.

Where subsurface flow dominates landscape evolution, considerable field investigation is needed of the conditions necessary for piping. It would be useful to map the groundwater potential field around spring heads and to study bedrock alteration in that zone. Hydrogeologic investigations combined with petrologic and structural analyses are needed to define the influence of various scales of rock heterogeneity on the groundwater flow field and therefore on the spacing and orientation of channels developed by spring sapping. Such field data should provide the
basis for a realistic numerical model of the flow and potential fields around an expanding system of spring heads until channel networks stabilize through competition for groundwater.

Very little attention has been paid to the question of how channels incise bedrock as the drainage basin is lowered through geologic time. The rate at which this lowering occurs is an important independent variable in models of hillslope and drainage basin evolution (Kirkby, 1971; Ahnert, 1976). The only formal investigation of a mechanism for bedrock erosion in channels appears to be that of Foley (1979), who used sand-blast abrasion theory to compute rates of bedrock incision by streams, and checked the results against interpretations of the Quaternary history of a river in Montana. The problem of channel incision needs more work of this kind and perhaps other studies of the synergistic role of chemical weathering with abrasion and in some mountain streams with scouring by debris flows. As with many other questions concerning drainage basin development, the mechanics of channel incision into bedrock is an important geomorphological problem that seems ripe for attack.

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