Stochastic Theory of Particle Trajectories through Alluvial Valley Floors

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ABSTRACT

Temporary storage of sediment within alluvial valley floors modulates the long-term transport of sediment through landscapes. The fate of weathering minerals or sediment-bound constituents in fluvial environments depends on the relative time scales of constituent degradation and particle residence time within valleys. Particles follow a set of trajectories through valley floors: some particles pass directly through the channel, reaching the basin outlet rapidly after being introduced to the fluvial system; others remain for long periods in deposits such as flood plains. Traditional sediment routing theory, based on the principle of sediment mass conservation along reaches of channel, does not account for exchanges of sediment with temporary sediment storage reservoirs outside the channel, such as flood plains, deltas, and alluvial fans. This article formalizes a theory that incorporates the role of such exchanges in the migration of sediment through river systems, by computing the probabilistic structure of particle trajectories through alluvial valley floors. Equations are developed for computing these trajectories from the sediment budget of a valley floor in steady state. Mathematical strategies for using such relationships to model transient storage conditions are proposed, and other potential model enhancements are discussed. The approach is illustrated using a hypothetical valley floor as an example. The theory can be used to examine rates of sediment overturn in valleys, map particle residence times, and account for the redistribution and decomposition of weathering minerals and particle-bound constituents. The theory has numerous potential management applications, some of which are discussed herein. The hypothetical example demonstrates that the probability distribution of particle residence times in the valleys of most alluvial rivers should be strongly right skewed.

Introduction

Sediment eroded from upland sources is often deposited in the alluvial floors of river valleys. Sediment can be deposited in a variety of storage reservoirs within the valley floor, including the channel bed, bars, flood plains, and deltaic deposits (fig. 1). However, routing sediment through rivers is usually treated as a one-dimensional mass conservation problem, in which sediment transport rates are estimated at channel cross sections and changes in storage are computed between them (Vanoni 1975). This approach has been valuable in a wide range of engineering and scientific applications, but it has at least two important limitations. First, it is widely understood that large quantities of sediment are stored outside river channels in deposits such as flood plains, and the annual rates of exchange between channel and flood plain can exceed the annual downstream flux (Meade 1982; Kesel et al. 1992; Dunne et al. 1998). Typical sediment routing models do not account for exchanges of sediment with such deposits or for the role of these deposits in modulating downstream sediment delivery. Second, the mass balance approach predicts changes in sediment storage along reaches of channel but cannot track individual particles through the valley floor. This important limitation makes it difficult to use traditional sediment routing models to predict the behavior of sediment-bound constituents in watersheds. Many pollutants, tracers, and nutrients enter fluvial sys-

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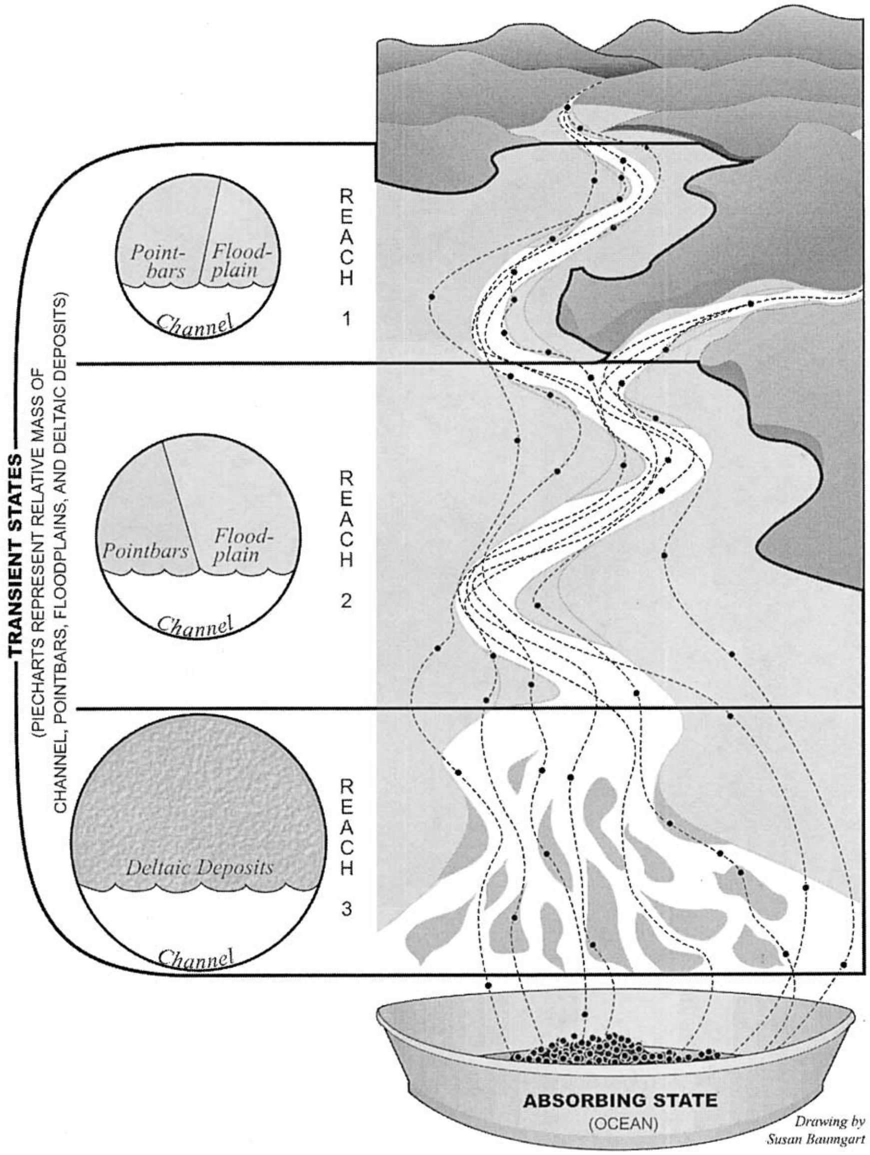

Figure 1. Schematic diagram illustrating the geomorphic context for using probability theory to analyze the trajectories of particles through alluvial valley floors. The state space of this example has eight transient states distributed among three reaches (channel bed, bars, and flood plains in reaches 1 and 2, and channel and delta deposits in reach 3) and a single absorbing state representing sediment delivery to the ocean. Reach boundaries are chosen at geomorphically significant locations, such as major tributary junctions or abrupt changes in valley morphology.

tems contained in or bound to particles. There are many applications in which it would be valuable to model their long-term redistribution and delivery in the alluvial environment.

One possible solution to both of these problems is to analyze the trajectories of particles as they move through a series of storage reservoirs in the valley floor, taking a Lagrangian, rather than the traditional Eulerian, approach to the sediment routing problem. Viewed over appropriate time and space scales, the trajectory of a particle through an alluvial valley floor is a random process consisting of episodes of transport separated by intervals of storage of varying length. Even if the rates of all the sediment transport and exchange processes in rivers were known precisely, the movement of a particular particle would still be a random process. For this reason, we followed the lead of Dietrich et al. (1982) and Kelsey et al. (1987), who proposed using probability theory to model the transport of sediment into and out of temporary sediment storage reservoirs.

Dietrich et al. (1982) presented equations to compute the residence time of sediment in steady state channel and flood plain reservoirs. They illustrated the procedure using dendrochronology of flood plain trees (Everitt 1968) and showed how the travel time of particles through such a deposit can be computed from the age distribution of sediment in that deposit. They emphasized that particle residence time in the active channel is always less than the residence time of sediment in the valley floor because of the possibility of sediment storage outside of the channel. They proposed that sediment exchanges among deposits of differing mobility could be expressed as transition probabilities.

Kelsey et al. (1987) elaborated this idea by characterizing the long-term movement of sediment through an alluvial valley floor as a discrete time Markov chain. They developed a Markov model of sediment transport in Redwood Creek, California, which routed sediment through three contiguous reaches of valley floor and into the Pacific Ocean. The authors computed the mean particle transit time to the ocean for particles starting in each of 12 temporary storage reservoirs. They modeled the changes in the volume of active, semiactive, inactive, and stable sediment reservoirs using measurements of reservoir volumes and estimates of bed load transport. This study demonstrated the feasibility of using probability theory to model long-term sediment movement through valleys.

Neither of these studies addressed the physical mechanisms by which sediment is exchanged with the flood plain and other temporary storage reservoirs. The purpose of this article is to formalize a process-based, probabilistic approach to sediment routing and to develop a general framework for parameterization using the sediment budget of a valley floor. We present equations for estimating trajectory probabilities and for using these probabilities to map particle residence times to evaluate the rate of sediment overturn in the valley floor and to examine the loci and duration of temporary particle storage. We also present a means of accounting for redistribution and degradation of particle-bound constituents. Some of the basic ideas presented were described in an earlier article (Malmon et al. 2002), which briefly demonstrated the potential of the approach for managers, using an example from Los Alamos, New Mexico. This article expands on and generalizes that study by presenting a more fully elaborated theoretical framework and develops the equations and procedures necessary to apply the theory in other field areas.

The article is organized as follows. In "Theoretical Development," we develop the theoretical framework, discuss model parameterization, and introduce a hypothetical example. "Analysis of the Model" contains equations for analyzing the model in steady state, on the basis of the theory of discrete time Markov chains. In "Discussion," we discuss the main limitations of the model in its current form and outline mathematical strategies that could be used to characterize three important aspects of natural fluvial systems for which equations are not presented in the current text: multiple particle size classes, non–steady state conditions, and the stochastic nature of forcing mechanisms.

Theoretical Development

The trajectory of a particle through an alluvial valley floor is a stochastic process influenced by rates of sediment transport, deposition, and remobilization. The stochastic model presented analyzes the trajectory of a hypothetical particle moving through a valley floor consisting of a finite number of sediment storage reservoirs in steady state. The steady state assumption requires that the mass of each of the deposits remains roughly constant over time. This assumption is approximately valid in many valleys over timescales relevant to the contamination and recovery of flood plains; the possibility of adapting the model to the transient case is discussed later.

A Markov chain is a stochastic process that takes on a finite number of values in which the transition from one state to the next is determined only by the current state of the process and not by its prior history (Ross 1997). If the future movement of a particle depends only on its present location and not its movement history, the process can be considered as a Markov chain. Because mathematical properties of Markov chains are simple and well understood, formulating the problem in such a way capitalizes on a well-established body of mathematical theory. The Markov chain is specified by (1) the state space, or the universe of values or states that the process can assume, and (2) the transition probabilities, which govern the movement of the process among the values in the state space.

State Space of the Model. Figure 1 schematically illustrates the nature of the state space in a valley floor, consisting of transient and absorbing states. Sediment stores such as the channel bed, bars, flood plains, and deltaic deposits are transient states, since particles reside in them temporarily. The downstream boundary represents an absorbing state, because a particle that enters it cannot return

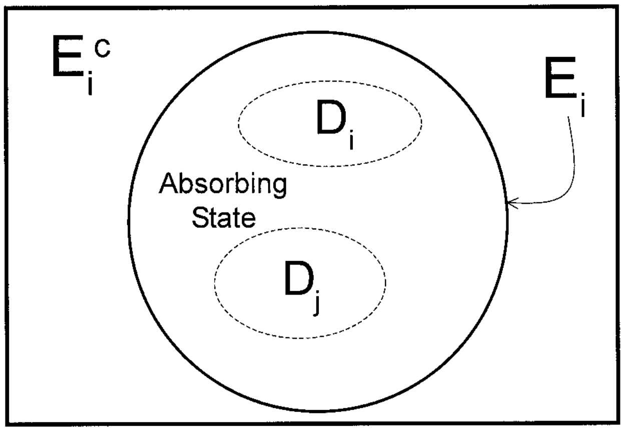

Figure 2. Venn diagram illustrating the derivation of transition probabilities for a model with two transient states, *i* and *j*, and a single absorbing state. The rectangle represents the universe of possible outcomes after an increment of time for a particle initially stored in transient state *i*. The probability of each outcome is equal to the proportion of the rectangle occupied by that outcome. Circle E_i is the event that the particle is eroded from *i*, and ovals D_i and D_j are the events that the particle is deposited in *i* and *j*, respectively, after first being mobilized from *i*.

to any of the other states. Additional absorbing states could be present where sediment is permanently removed from the valley floor by tectonic movements or by engineering, a capability that could allow treatment of processes such as sediment deposition within a subsiding alluvial basin or the impact of in-stream gravel extraction on the migration of particles through rivers. For simplicity, the model outlined contains only one absorbing state (sediment transport past the downstream boundary); thus, there are $b + 1$ states in the state space (Ω) of the process. We denote the set of transient states by the letter B, the set of absorbing states by A, and the entire state space by Ω. Set B contains b temporary sediment stores, and set A contains a absorbing states.

The valley floor can be divided into reaches to account for downstream variations in sediment storage and exchange rates. Reaches are delineated on the basis of major tributary junctions, changes in valley morphology, or at other points where sediment and constituent flux are of interest (fig. 1). Within each reach, the active portion of the valley floor is treated as a set of discrete transient states. Within each state, all particles are equally susceptible to future erosion, sediment transport, and deposition. The approximation of equal mobility within each transient state is central to several of the equations presented. Therefore, storage units must be delineated in such a way as to ensure that this is a reasonable approximation over some relatively long timescale on the order of decades or longer. Examples of such storage elements include the channel bed, bars, flood plain units, or geographical subsets of these deposits.

The property of equal mobility for each transient state is a fundamental assumption of the mathematical treatment developed in this article. The validity of the equal mobility assumption depends entirely on a realistic delineation of the state space, which must be based on a solid, field-based conceptual model of the sediment budget. Several geologic and geomorphic factors, such as particle size and stratigraphy, must be considered with respect to this assumption, which states that, at least in an approximate way, all the particles within each transient state are equally susceptible to future mobilization, transport, and deposition.

Note that the equal mobility assumption does not require that sediment in a reservoir be well mixed. For example, the particles in a vertically accreting flood plain that erodes by lateral bank erosion could be considered equally mobile, even though the flood plain could contain distinct layers of sediment with different particle size and age characteristics. An erosion event such as a bank collapse would mobilize a sample of the entire stratigraphic section, including layers of old and young material. The issue of treating particle size variability within the context of the equal mobility assumption is elaborated in "Discussion."

Let r denote a particular reach and B_r denote the subset of the transient state space located within it (that is, $B_r \subset B$). In the following discussion, we assume that all the deposits within reach r can be reached from one another within a single time increment. However, particles stored downstream of r cannot reach any of the elements in B_r.

Transition Probabilities and the Transition Matrix. A particle in a transient state or geomorphic unit i has a fixed probability $p_{ij} \geq 0$ of moving to state j after a unit time. These transition probabilities are controlled by the rates of sediment transfer within and through the valley floor. Kelsey et al. (1987) assigned transition probabilities on the basis of a qualitative ordering of the relative importance of the various processes in the sediment budget. Here we present a systematic strategy for computing the transition probabilities directly from an estimate of the sediment budget of the valley floor.

Each transition consists of two distinct events: (1) the erosion event E_i that causes the particle to

Table 1. States in the State Space for Hypothetical Valley Floor

State	
1	Reach A channel
2	Reach A flood plain
3	Reach B channel
4	Reach B flood plain
5	Reach C channel
6	Reach C flood plain
7 (x)	Absorbing state, transport past downstream boundary

be mobilized from unit i and (2) the deposition event D_j that places the particle in unit j. The Venn diagram in figure 2 illustrates the derivation of transition probabilities for a particle stored in transient state i within a valley floor consisting of two transient states, i and j (in the same reach) plus one absorbing state.

If the particle is currently residing in unit i, the task is to compute the probabilities that the particle will reside in unit i, unit j, and the absorbing state after a time increment. The sum of these three probabilities is 1 because these are the only three outcomes in this simple model. The rectangle in figure 2 represents the universe of possible outcomes and has an area of 1. The proportion of the rectangle area occupied by a given outcome corresponds to the probability of that outcome. There are four distinct regions in the Venn diagram in figure 2, but two of these represent trajectories that result in the same outcome—the particle remaining in unit i—and thus, there are only three outcomes and three transition probabilities.

For a particle to move from unit i to unit j, it must first be eroded from i (event E_i in fig. 2) and then deposited in j (the conditional event D_j in fig. 2). In general, the transition probability per time p_{ij}, where $i \neq j$, is equal to the fraction of the entire sample space occupied by event D_j:

$$p_{ij} = P(E_i)P(D_j|E_i). \tag{1}$$

Equation (1) is a rearrangement of Bayes' formula (e.g., Ross 1997, p. 14).

The particle can remain in i either by not being mobilized (event E_i^c, the complement of E_i,) or by being mobilized and then redeposited in i (oval D_i). The transition probability p_{ii} is the sum of probabilities of these two outcomes (see fig. 2):

$$\begin{aligned} p_{ii} &= P(E_i^c) + P(E_i)P(D_i|E_i) \\ &= [1 - P(E_i)] + P(E_i)P(D_i|E_i). \end{aligned} \tag{2}$$

If a particle is mobilized within the valley floor and not redeposited in any of the b transient states, it reaches the absorbing state, whose index is x (i.e., it leaves the system at the downstream boundary). In figure 2, the probability that a particle starting in unit i is transported directly out of the model system at the downstream boundary, p_{ix}, is the fraction of area inside E_i but not occupied by D_i or D_j. Generalizing this principle to a system containing an arbitrary number, b, of transient states and a single absorbing state at the downstream boundary, we find that the probability per time that a particle exits the valley floor is

$$p_{ix} = P(E_i)\left[1 - \sum_{j=1}^{b} P(D_j|E_i)\right]. \tag{3}$$

Using equations (1)–(3), we can compute the transition probabilities from $P(E_i)$, the erosion probabilities, and $P(D_j|E_i)$, the deposition probabilities, which are determined using the sediment budget.

Erosion Probabilities. For transient state i, the probability $P(E_i)$ of any particle being mobilized per unit time is the inverse of the mean residence time of sediment in that deposit. Dietrich et al. (1982) presented equations for computing this residence time from the age distribution of sediment stored in a deposit, measured from dendrochronology applied to flood plain trees. However, in practice these data are not available for every sediment reservoir. If all particles within a reservoir can be considered to be equally mobile (a requirement of the geomorphic delineation of the state space, as discussed previously), then the erosion probability per unit of time is the mass rate of erosion of deposit i divided by the total mass of that deposit:

$$P(E_i) = \frac{Q_{E_i}}{m_i}, \tag{4}$$

where Q_{E_i} is the erosion rate of deposit i (mass/time) and m_i is the mass of unit i. Equation (4) assumes that all the particles in unit i are equally susceptible to erosion. The transient states must be defined in such a way as to ensure this assumption is a reasonable approximation, as discussed previously.

In some cases, it might be necessary to separate portions of geomorphic units to improve the validity of the equal mobility assumption for computing erosion probabilities. For example, within a given reach, the flood plain may be subdivided into areas near the channel and farther from the channel, since particles closer to the channel have a higher probability of being eroded. However, as both a deposition rate and an erosion rate must be estimated

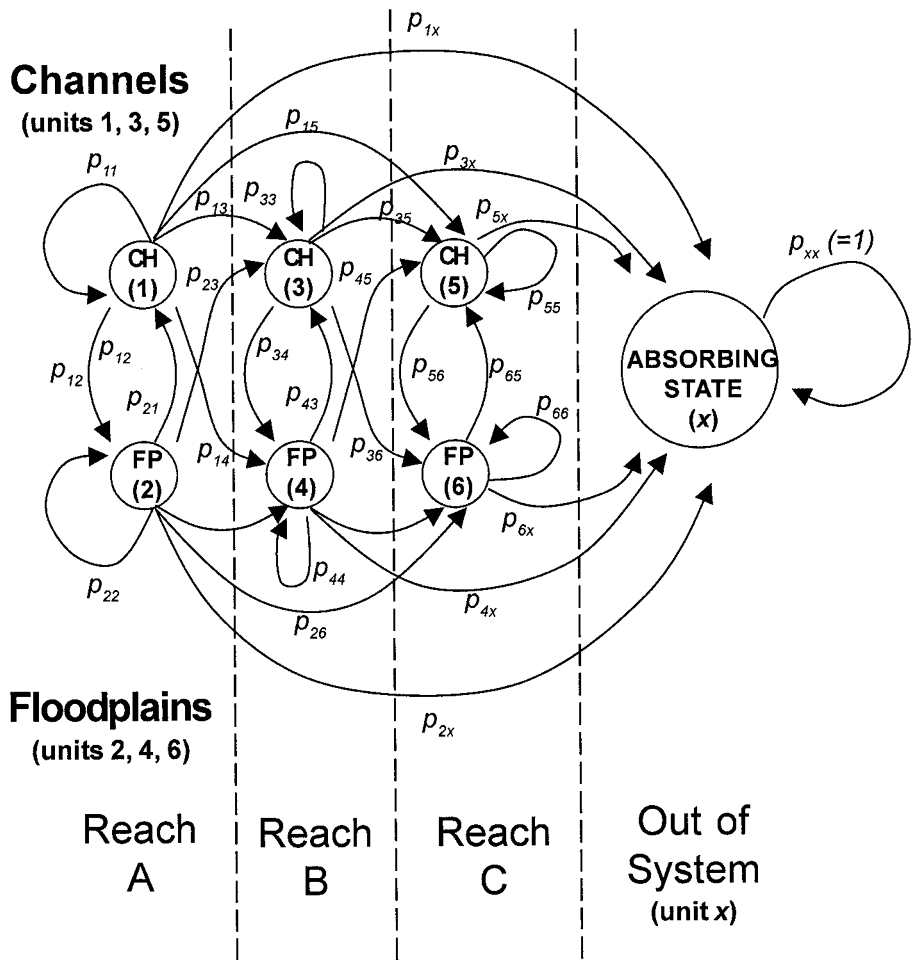

Figure 3. Schematic diagram of a Markov model for a hypothetical valley floor divided into three reaches, each containing a channel and flood plain. In this example, a particle can be deposited in the channel or flood plain or transported into the absorbing state. Odd-numbered states are the channel units, even-numbered states are the flood plains, and state x is the absorbing state. The transition probabilities are computed from the sediment budget (table 1) and equations (1)–(8). The matrix containing the computed transition probabilities is presented in table 3.

for each transient state, increasing the size of the state space requires a corresponding increase in the amount of data required to parameterize the model. The level of detail and realism represented by an application of the model must be weighed against the availability and reliability of sediment budget data.

Deposition Probabilities. If both i and j are located within reach r, then the conditional probability that a particle will be deposited in j, given that it has eroded from i, is

$$P(D_j|E_i) = \frac{Q_{D_j}}{Q_{O_r} + \sum_{k \in B_r} Q_{D_k}}, \qquad (5)$$

where Q_{D_j} is the mass rate of sediment deposition into deposit j (mass/time), B_r is the portion of the transient state space B that is located in reach r, and Q_{O_r} is the sediment flux out of reach r at its downstream end. The summation $\sum_{k \in B_r} Q_{D_k}$ represents the total rate of sediment deposition into all the units located within reach r (including state j). A particle in transport within reach r (whether it entered from upstream or from external sources or was eroded from one of the units located in that reach) will either be deposited in one of the reservoirs in B_r or will exit the reach at its downstream boundary. The denominator in equation (5) equals the total mass of sediment in transport within reach r, and the probability $P(D_j|E_i)$ is the mass fraction of that sediment that is deposited in j.

If the particle is not deposited in any of the units in reach r, it enters the downstream reach $r+1$. The probability $P(O_r|E_i)$ that a particle leaves reach r given that it was eroded from deposit i in reach r is

$$P(O_r|E_i) = \frac{Q_{O_r}}{Q_{O_r} + \sum_{k \in B_r} Q_{D_k}}. \quad (6)$$

If j is a transient state located in the reach immediately downstream of the reach where i is located, then in order for a particle to move from i to j in a unit time, the following three events must occur: (1) the particle must be eroded from i (in reach r), (2) the particle must be transported out of reach r, and (3) the particle must be deposited in j (in reach $r+1$). In this case, the conditional probability that a particle is deposited in j, given that it eroded from i, is the intersection of events 2 and 3:

$$\begin{aligned} P(D_j|E_i) &= P(O_r|E_i) \cap P(D_j|O_r) \\ &= P(O_r|E_i)\frac{Q_{D_j}}{Q_{O_{r+1}} + \sum_{k \in B_{r+1}} Q_{D_k}} \end{aligned} \quad (7)$$

for $i \in B_r$ and $j \in B_{r+1}$, where the conditional probability $P(O_r|E_i)$ is determined from equation (6).

In general, if i is located in reach r and j is located in an arbitrary reach n downstream of reach r, a particle must consecutively enter and leave each intermediate reach and eventually deposit in unit j. The probability of this occurring tends to decrease with increasing distance downstream, since the deposition probability is the product of an increasing number of terms less than 1:

$$\begin{aligned} P(D_j|E_i) = {} & P(O_r|E_i)P(O_{r+1}|O_r) \ldots \\ & P(O_{n-1}|O_{n-2})P(D_j|O_{n-1}) \end{aligned} \quad (8)$$

for $i \in B_r$ and $j \in B_n$, where B_n is the set of geomorphic units or transient states located within reach n.

In summary, all the transition probabilities can be computed from the sediment budget, which consists of (1) the erosion and deposition rates, Q_{E_i} and Q_{D_i}, of each geomorphic unit i; (2) the sediment flux Q_{O_r} at the downstream boundary of each reach r; and (3) the mass m_i of each of the storage reservoirs. The transition probabilities are arranged in a transition matrix $\mathbf{P} = \{p_{ij}\}$:

$$\mathbf{P} = \begin{pmatrix} p_{11} & \cdots & p_{1b} & p_{1x} \\ \vdots & \ddots & & \\ p_{b1} & & p_{bb} & p_{bx} \\ 0 & 0 & \cdots & 1 \end{pmatrix}, \quad (9)$$

where x is the index of the absorbing state and b is the total number of transient states in the transient state space, B. The final row contains the transition probabilities for particles starting in the absorbing state and indicates that particles that have already been transported out of the system remain out of the system with probability 1. The row sums in $\mathbf{P}$ must all equal 1 to account for all possible outcomes for a particle starting in unit i. All the information for computing particle trajectories in steady state valleys is contained in the transition probability matrix.

Hypothetical Example. To illustrate the model, a hypothetical alluvial valley floor is divided into three reaches, each containing a channel sediment store and a flood plain store (table 1). For simplicity, all the particles entering the valley are similar and suspendible by flood flow. This situation might represent a well-sorted sand-bed river with sandy banks. Adaptations depicting a wider range of natural sorting processes are discussed later.

During the course of a year, particles are exchanged between the channel and flood plain and transported downstream. Figure 3 shows all the transitions possible in a given year. In this example, all the downstream and local (i.e., within the same reach) states are accessible from each transient state. The transition matrix is

$$\mathbf{P} = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} & p_{1x} \\ p_{21} & p_{22} & p_{23} & p_{24} & p_{25} & p_{26} & p_{2x} \\ 0 & 0 & p_{33} & p_{34} & p_{35} & p_{36} & p_{3x} \\ 0 & 0 & p_{43} & p_{44} & p_{45} & p_{46} & p_{4x} \\ 0 & 0 & 0 & 0 & p_{55} & p_{55} & p_{5x} \\ 0 & 0 & 0 & 0 & p_{65} & p_{66} & p_{6x} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad (10)$$

where the odd subscripts represent channel units increasing downstream, the even subscripts represent flood plain units, and x is the absorbing state (fig. 3).

A hypothetical sediment budget for this system, with figures reasonable for a small stream, is presented in table 2. The total amount of sediment stored within each reach is 21 million metric tons (T), including 10^6 T of channel-stored sediment and 20×10^6 T in the flood plain. The sediment flux

Table 2. Sediment Budget of Hypothetical Valley Floor

	Reach A	Reach B	Reach C
Channel mass ($\times 10^6$ T)	1	1	1
Flood plain mass ($\times 10^6$ T)	20	20	20
Downstream sediment flux ($\times 10^6$ T/yr)	2	2	2
Channel erosion/deposition rate ($\times 10^6$ T/yr)	.5	.5	.5
Flood plain erosion/deposition rate ($\times 10^6$ T/yr)	1	1	1

Note. T = tons.

(Q_O) through each reach is 2×10^6 T/yr. Half the sediment stored in the channel in a given year is mobilized by channel erosion (0.5×10^6 T/yr). Five percent of the flood plain sediment is mobilized by bank erosion each year (10^6 T/yr; table 2), and this material is replaced each year by flood plain sedimentation. All the entries in the transition probability matrix (eq. [10]) can be computed from the sediment budget in table 2 using equations (1)–(8). For example, the probability of a particle moving from the flood plain in reach A (unit 2) to the channel in reach C (unit 5) in any one year is:

$$\begin{aligned} p_{25} &= P(E_2)P(O_A|E_2)P(O_B|O_1)P(D_5|O_2) \\ &= \left(\frac{10^6}{20 \times 10^6}\right) \\ &\quad \times \left(\frac{2 \times 10^6}{2 \times 10^6 + 0.5 \times 10^6 + 10^6}\right) \\ &\quad \times \left(\frac{2 \times 10^6}{2 \times 10^6 + 0.5 \times 10^6 + 10^6}\right) \\ &\quad \times \left(\frac{0.5 \times 10^6}{2 \times 10^6 + 0.5 \times 10^6 + 10^6}\right) \\ &= 0.0023. \end{aligned} \tag{11}$$

The numerical subscripts on E and D refer to the numbered units in the transient state space B (with odd numerals representing channel units and even units representing flood plains, as in fig. 3), whereas the alphabetical subscripts on O refer to the name of the reach. The remainder of the entries in the transition probability matrix (table 3) are computed in the same way. Note that all the row sums in table 3 equal 1, with minor deviations due to rounding errors.

Analysis of the Model

Definition of Terms: Transit Time, Flushing Time, and Residence Time. We adopt the relevant terminology of Dietrich and Dunne (1978), Dietrich et al. (1982), and Kelsey et al. (1987) wherever possible. The characteristic residence time of sediment in a deposit is the expected amount of time a particle will remain in that deposit before being remobilized. The residence time of a deposit is thus the inverse of the erosion probability of that deposit (eq. [4]). Dietrich and Dunne (1978) estimated a residence time (per meter of valley length) for channel and flood plain deposits by dividing the volume per meter of each reservoir by the volumetric bed load flux. For the channel bed, this definition of residence time can be interpreted as the product of the reach length with the inverse of the velocity of bed load sediment through the channel. For the flood plain, it is more difficult to interpret this definition in terms of physical processes. Kelsey et al. (1987) slightly modified this definition, dividing the bed load flux by sums of reservoir volumes to increase residence times for less active deposits. Regardless of the denominator, the bed load flux in the numerator is probably not a realistic measure of the rate at which sediment is mobilized from storage, particularly for deposits such as flood plains. Thus, estimates of residence time for fluvial sediment reservoirs have been somewhat arbitrary and amounted to "an index of the size of the reservoir" (Kelsey et al. 1987, p. 1742) rather than a quantitative, process-based definition. A more general definition of the expected particle residence time (for a reservoir in steady state) is the inverse of the erosion probability in equation (4): the mass of a particular deposit divided by the erosion rate of that deposit. Note that this definition of residence time does not specify the amount of time particles will ultimately spend in a deposit because particles can be mobilized and redeposited in the same reservoir multiple times.

The residence time of a deposit is distinguished from the transit time for a particle in that deposit, which we define here as the length of time a particle takes to reach an absorbing state. This definition is different from the use of the term by Dietrich et al (1982), in which the transit time function referred to the cumulative age curve for sediment leaving a particular reservoir (their description did not include an absorbing state). Some particles will be quickly mobilized and transported rapidly out of the valley, while others will remain in storage for long periods or will be repeatedly stored in downstream deposits. Thus, the sediment in each transient state or sediment storage reservoir exhibits a probability distribution of particle transit times. The mean of this distribution is what Kelsey et al. (1987) called the flushing time of the deposit, which is the expected amount of time a particle starting in that deposit will spend in the valley floor.

Probability That a Particle Will Be in a Given Place at a Given Time. Matrix **P** (eq. [9]) contains the transition probabilities p_{ij} for particle movement during a single time increment. The probability structure of particle transitions at an arbitrary time t can be computed using the Chapman-Kolmogorov equations, a fundamental theory for discrete time Markov chains (Ross 1997). Let $\mathbf{P}(t)$ denote the t-step transition matrix, which contains the probabilities $p_{ij}(t)$ that a particle starting in state i will reside in state j after exactly t years. The theory states that

$$\mathbf{P}(t) = \mathbf{P}^t. \tag{12}$$

In other words, the t-step transition matrix is equal to the t^{th} power of the single-step transition matrix (according to the definition of powers for a square matrix). Thus, the probability that a particle will reside in j at time t, given it started in i, is the entry in the ith row and jth column of $\mathbf{P}^t$.

The Chapman-Kolmogorov equations define the probability structure of future particle trajectories for all the sediment currently stored in the valley floor and can be used in a variety of applications. For example, if mining waste containing heavy metals was introduced into a river channel or other reasonably well-mixed sediment reservoir i (without significantly affecting the total volume and therefore the steady state condition of the valley floor), and then the releases ceased, it is straightforward to compute the distribution of the metal at any subsequent time t: the proportion of the introduced metal stored in every state in the state space at time t is the ith row of $\mathbf{P}^t$.

Particle Transit Times. The particle transit time is the time a particle takes to reach an absorbing state, starting from some initial deposit i. Some particles will exit the system rapidly, whereas others will be stored repeatedly within intermediate storage for long periods of time. Kelsey et al. (1987) showed that the mean particle transit time for each temporary storage reservoir could be easily computed using the fundamental matrix (which is discussed further later). They also presented an equation (eq. [10] in their article) that can compute the variance in transit times for particles starting in each transient state.

If transit times were normally distributed, the entire distribution of particle transit times could be specified from the mean and variance. Dietrich et al. (1982) hypothesized that particle transit times for sediment transport through river valleys are probably not normally distributed, so the mean transit time (i.e., the flushing time) may be a poor indicator for the bulk of sediment in a given storage reservoir. They pointed out that, in order to address questions relating to chemical and physical changes to which sediment is subjected while traveling through a valley, "one must attempt to define the transit-time distribution" (Dietrich et al. 1982, p. 20).

It is possible to derive the probability density function of transit times for each reservoir using the Chapman-Kolmogorov equations. Let $\mathbf{P}^t$ be the t-step transition matrix for a system with only one absorbing state, x; the one-step transition matrix is arranged as in equation (9). The proportion of particles originating in transient state i that have a transit time of t years is equivalent to the probability that any particular particle reaches the absorbing state in exactly t years. This probability is

$$g_i(t) = \mathbf{P}^t_{ix} - \mathbf{P}^{t-1}_{ix}, \tag{13}$$

where $g_i(t)$ is the transit time probability density at time t for sediment in unit i at time 0 and $\mathbf{P}^t_{ix}$ denotes the entry from the ith row and last column of the matrix **P** raised to the t^{th} power. Figure 4 shows the transit time distributions computed from the transition probability matrix in table 3 using equation (13). The transit time distributions in figure 4 account for repeated sediment storage in transient states during particle trajectories and can be interpreted as the probability density function of particle residence time in the valley floor. All six probability distributions are strongly right skewed.

The two plots have different timescales, reflecting the much higher mobilization probabilities (lower residence times) associated with channel-stored sediment. In figure 4*a*, the modal transit time for the channel-stored sediment is exactly 1 yr, whereas modal transit times for flood plain sediment vary from 3 yr (reach C) to 9 yr (reach A; fig. 4*b*). Mean transit times for channel-stored sediment are much longer than they would appear from figure 4*a*; this is because the tails of the distributions in figure 4 extend to infinity. The long tails,

Table 3. One-Step Transition Probabilities for Hypothetical Valley Floor

Particle location at time t (state i)	Particle location at time $t + 1$ (state j)						
	1. Reach A channel	2. Reach A flood plain	3. Reach B channel	4. Reach B flood plain	5. Reach C channel	6. Reach C flood plain	7 (x). Absorbing state
1. Reach A channel	.571	.143	.041	.082	.023	.047	.093
2. Reach A flood plain	.0071	.964	.0041	.0082	.0023[a]	.0047	.009
3. Reach B channel	0	0	.571	.143	.041	.082	.163
4. Reach B flood plain	0	0	.0071	.964	.0041	.0082	.16
5. Reach C channel	0	0	0	0	.571	.143	.286
6. Reach C flood plain	0	0	0	0	.0071	.964	.029
7 (x). Absorbing state	0	0	0	0	0	0	1

Note. Probabilities are that a particle starting in i will be in j after a single increment of time.
[a] Derivation of probability p_{25} is demonstrated by equation (11) in the text.

not visible in figure 4*a*, primarily reflect the particles initially stored in the channels that temporarily settle in flood plains, where they may remain for hundreds or thousands of years before being remobilized. In general, sediment has a low probability of being deposited farther from the channel, where it is least likely to be remobilized. This leads to long tails on such distributions and suggests a generalizable hypothesis that strongly right-skewed transit time distributions are characteristic of sediment reservoirs in alluvial valleys.

Computing the transit time distributions from equation (13) could be useful in many scientific and management applications. These distributions quantify the mechanisms by which the various geomorphic reservoirs regulate sediment delivery in fluvial systems. Sensitivity analyses involving transit time distributions could be used to predict the influence of environmental conditions (which control the transition probabilities) on the rate and nature of sediment delivery from alluvial valley floors. These results are applicable not only to the flushing of contaminated sediment from an alluvial valley but also to interpretations of other geomorphological records in alluvium, such as the suite of fission-track ages in minerals released by erosion after a pulse of mountain building, which may partially reflect the distribution in particle transit times from the source area to the location where the sediments were sampled.

Mean Time Spent in Transient States. In some applications, it may be useful to estimate how long particles will spend in each of the downstream storage reservoirs before entering the absorbing state. Let s_{ij} denote the expected time that a particle starting in i will spend within transient state j before reaching the absorbing state. Let $\mathbf{S}$ denote the matrix of values s_{ij} for all $i, j \in B$ (i.e., a $b \times b$ matrix for all the transient states). Matrix $\mathbf{S}$ is called the fundamental matrix and is computed from

$$\mathbf{S} = (\mathbf{I} - \mathbf{P}_B)^{-1}, \quad (14)$$

where $\mathbf{P}_B$ specifies the submatrix of $\mathbf{P}$ containing only the transition probabilities from transient states to transient states and $\mathbf{I}$ is the identity matrix with the same dimensions as $\mathbf{P}_B$ (Resnick 1992, p. 106). The expected length of time a particle starting in i will spend in each of the transient states before reaching the absorbing state is the ith row of matrix $\mathbf{S}$. The sum of the ith row of $\mathbf{S}$ is the expected amount of time for a particle starting in i to reach the absorbing state, as pointed out by Kelsey et al. (1987). The fundamental matrix computed for the hypothetical example is presented in table 4. The matrix shows that, given the sediment budget in table 2, particles spend most of their time in the valley floor within flood plain deposits. This statement is especially true for sediment that starts in flood plain deposits, where, on average, particles remain for approximately 30 yr before being mobilized initially and less than 25 yr in subsequent storage. In long reaches of large rivers (conditions for which the sediment budget in this example is not realistic), the probability of fine particles reaching the outlet without interacting with the flood plain would become smaller. In this case, both the expected duration in the flood plain and the proportion of the transit time spent in flood plain storage would be even greater.

In applications where sediment-bound constituents decay via chemical, physical, or biological processes, equation (14) can be used to evaluate the time available for constituent processing. The fate of such constituents is ultimately determined by the relative timescales of the chemical decay process and the residence time of particles in different

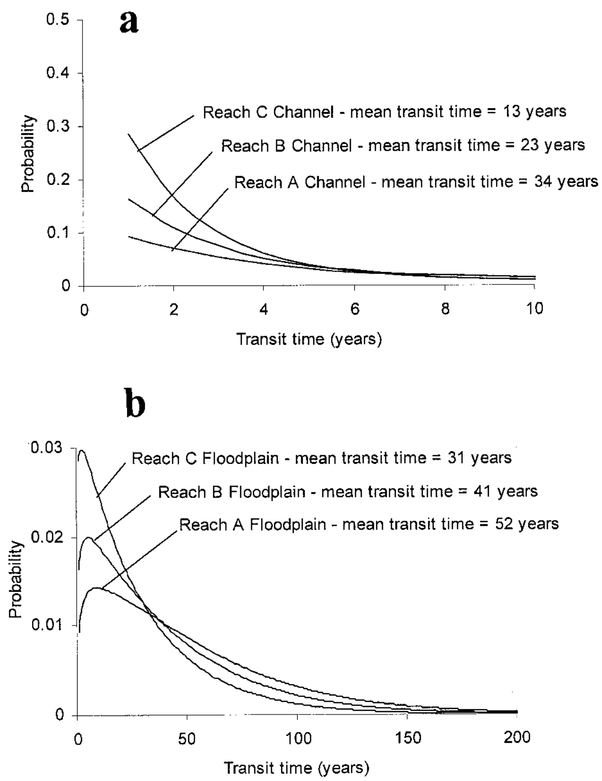

Figure 4. Probability distributions of particle transit time (i.e., particle residence time within the valley floor) in the hypothetical example. *a*, Sediment initially stored in the channel units (states 1, 3, and 5; fig. 3). Probability distributions extend to infinity, and the low but finite probability of very long transit times account for high mean transit times relative to the modal value (1 yr for all three distributions). *b*, Sediment initially stored in flood plain units (states 2, 4, and 6).

deposits. The fundamental matrix estimates the amount of time particles are expected to spend in each reservoir. Quantifying such properties in rivers could contribute to the analysis of problems related to weathering or constituent processing, particularly if the reservoirs have different oxidizing or pH conditions.

Another potential application of equation (14) relates to the hypothesis that downstream fining of bed sediment in some gravel rivers is controlled by weathering during long periods of particle storage in the flood plain (Jones and Humphrey 1997). According to this hypothesis, particles stored in flood plains develop weathering rinds whose thicknesses are a function of the duration of sediment storage in the flood plain. During intermittent episodes of particle transport along the channel bed, these rinds are quickly removed but subsequent fining by abrasion is limited. Using equation (14), we find that it is possible to compute the amount of time an average particle will spend in the flood plain per kilometer of travel distance along the channel. A simple particle weathering function could be developed by sampling sediment of varying age and measuring the abrasion rate of each sample with tumbling mill experiments (this was attempted by Jones and Humphrey 1997). Then the fundamental matrix could be used to determine the amount of time available for weathering in the flood plains and, thus, to quantify the role of flood plain weathering in downstream fining in gravel rivers.

Time Required for Evacuation of the Valley Alluvium. Next, we evaluate the timing of cumulative delivery of fluvial sediment from a valley floor in steady state. Let $h_i(t)$ be the mass of sediment entering the absorbing state at time t (mass/time) that originated in transient state i at time 0. Then

$$h_i(t) = g_i(t) \times m_i, \tag{15}$$

where m_i is the mass of sediment in deposit i and $g_i(t)$ is the transit time probability density, computed from equation (13). The cumulative mass flux of sediment into the absorbing state over time is computed by adding the contributions from each of the original deposits and integrating over time:

$$H(t) = \int_0^t \left[\sum_{i=1}^{b} h_i(t)\right] dt, \tag{16}$$

where $H(t)$ is the cumulative mass flux of valley-stored sediment into the absorbing state. In this context, valley-stored sediment refers to all the particles stored in the valley at time 0. Imagine painting all the particles in each transient state a different color at time 0. Equation (15) quantifies the flux of sediment of each color into the absorbing state over time, and equation (16) computes the cumulative mass of painted particles entering the absorbing state after time 0. The value $H(t)$ is not the same as the sediment flux into the absorbing state because it does not account for future sources of sediment from sources outside the valley floor (i.e., unpainted particles entering from hillslopes and tributaries). In a steady state valley, the total sediment flux into the absorbing state will remain constant through time, but the proportion of valley-derived sediment relative to external sediment will decrease.

Using equations (15) and (16), one can compute the rate at which sediment in the valley floor is evacuated and replaced with new sediment. For ex-

Table 4. Expected Particle Transit Times for Particles Initially Stored within Hypothetical Valley Floor

Initial location of particle	Expected duration in transient states (yr): 1. Reach A channel	2. Reach A flood plain	3. Reach B channel	4. Reach B flood plain	5. Reach C channel	6. Reach C flood plain	Expected transit time through valley floor
1. Reach A channel	2.5	10	.5	10	.5	10	34
2. Reach A flood plain	.5	30	.5	10	.5	10	52
3. Reach B channel	0	0	2.5	10	2.5	10	23
4. Reach B flood plain	0	0	.5	30	.5	10	41
5. Reach C channel	0	0	0	0	2.5	10	13
6. Reach C flood plain	0	0	0	0	.5	30	31

Note. Expected particle durations in transient states given by the fundamental matrix $\mathbf{S} = (\mathbf{I} - \mathbf{P}_B)^{-1}$, where $\mathbf{P}_B$ is the 6 × 6 matrix of transition probabilities among transient states and $\mathbf{I}$ is the identity matrix. Expected transit time through valley floor is the sum of expected durations in all transient states.

ample, the replacement time for half the sediment in the valley can be computed by solving equations (15) and (16) for $H(t)/\sum_{i\in B} m_i = 0.5$. The rate of overturn of sediment for the hypothetical valley system is plotted in figure 5. In this example, 50% of the sediment initially stored in the valley floor is evacuated within 21 yr, 90% in 91 yr, and 95% in 116 yr. Experimentation using field data from upper Los Alamos Canyon, New Mexico (Malmon 2002), showed that the time required to evacuate 90% of the sediment in the valley floor (denoted T_{90}) is useful as an index to compare the relative rates of sediment overturn under various conditions.

The amount of time required for overturn of sediment in an alluvial valley is scale dependent (i.e., evacuation rates are longer in a long reach than in a short reach and longer for wide flood plains than for narrow flood plains). The rate of sediment evacuation also depends on the rates of geomorphic processes, with more rapid process rates in general leading to more rapid sediment evacuation from the valley floor. In the hypothetical valley floor, doubling the size of the sediment reservoirs and reducing the rates of geomorphic processes by 50% leads to an increase in T_{90} from 91 yr to 365 yr (fig. 5).

In general, one would expect that particle transit times and rates of sediment evacuation would be longer for large, lowland rivers with wide flood plains and relatively low rates of sediment exchange with the flood plain. In contrast, particle transit times and rates of sediment evacuation are more rapid in steep, narrow valleys with relatively small flood plains and rapid sediment exchange.

Disposition of Particles Entering from outside the Valley. The preceding discussion concerned the fate of particles residing within the valley floor at time 0. However, the framework presented earlier can also be used to analyze the disposition of particles entering the system from upstream or from lateral hillslope or tributary sources. If we assume the externally derived sediment is physically similar to the sediment already stored in the valley floor (the chemistry may be different, as illustrated in the next section), the only additional calculation required is to partition the externally derived sediment among the transient states and the absorbing state during the first increment of time. The subsequent fate of those particles is the same as that of the sediment initially stored in the valley floor.

The disposition of the externally derived sediment in the first time increment can be computed by partitioning the influx according to the distribution of deposition probabilities downstream of the source area of interest, using equations (5)–(8) (in this case, the probabilities are conditioned on the fact that the particle entered the system from

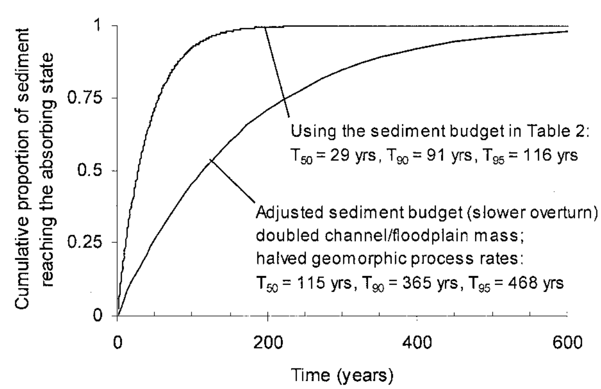

Figure 5. Modeled rate of evacuation of sediment from the valley floor. Given the hypothetical sediment budget in table 2, half the sediment stored in the valley at time 0 reaches the absorbing state within 29 yr, 90% within 91 yr, and 95% within 116 yr. Doubling the size of the sediment storage reservoirs and halving the rates of each geomorphic process (downstream sediment flux, exchange with the channel, and exchange with the flood plain) lead to a slower rate of sediment evacuation.

an external source rather than eroded from a deposit within the system). For example, let $\boldsymbol{P}_{\text{ext}}$ be a $1 \times b$ row vector containing the probabilities of deposition into the b transient states, computed using equations (5)–(8) for the hypothetical valley floor. The fourth entry in $\boldsymbol{P}_{\text{ext}}$ is the probability that a particle entering the system from upstream immediately enters the flood plain in reach B (state 4, fig. 3):

$$\begin{aligned}\boldsymbol{P}_{\text{ext}}(4) &= P(O_A|\text{particle entered from upstream}) \times P(D_4|O_A) \\ &= \left(\frac{2 \times 10^6}{2 \times 10^6 + 0.5 \times 10^6 + 10^6}\right) \times \left(\frac{10^6}{2 \times 10^6 + 0.5 \times 10^6 + 10^6}\right) \\ &= 0.16.\end{aligned} \tag{17}$$

Note that $\boldsymbol{P}_{\text{ext}}$ will sum to less than 1, with $1 - \Sigma\boldsymbol{P}_{\text{ext}}$ being the proportion of sediment entering from upstream that reaches the absorbing state in the first year.

Suppose we want to compute the expected transit time of a particle that enters the valley floor from upstream, tt_{upst}. This is the weighted average of the expected transit times for all the deposits, weighted according to the proportion of the influx that initially enters in each of the six transient states and the absorbing state:

$$tt_{\text{upst}} = \boldsymbol{P}_{\text{ext}}\boldsymbol{S}_{\text{sum}} + \left(1 - \sum \boldsymbol{P}_{\text{ext}}\right), \tag{18}$$

where $\boldsymbol{S}_{\text{sum}}$ is a $b \times 1$ column vector containing the row sums of the fundamental matrix (eq. [14], or the last column in table 4) and $\boldsymbol{P}_{\text{ext}}\boldsymbol{S}_{\text{sum}}$ is computed according to the rules of vector multiplication. The second term on the right side of equation (18) will always be small (less than 1 yr) and accounts for the proportion of the sediment that immediately enters the absorbing state. In the hypothetical valley, tt_{upst} is 31.7 yr, with approximately 19% of the upstream sediment reaching the downstream boundary in less than 1 yr. Computing the disposition of externally derived particles can be particularly useful when the chemical quality of sediment changes because of some natural or anthropogenic perturbation, as is illustrated with an example in the following section.

Fate and Decomposition of Particle-Bound Constituents. Even if a valley floor can be considered to be in steady state with respect to sediment storage, the constituent load of this sediment can vary over time. The probabilistic approach for routing sediment through valley floors is useful for tracking the redistribution and decay of sediment-bound constituents such as tracers and contaminants. Let $w_i(t)$ be the inventory of a stable constituent in storage reservoir i at time t. Then

$$w_i(t) = m_i c_i(t), \tag{19}$$

where $c_i(t)$ is the concentration of the constituent on sediment in i at time t. If $\boldsymbol{W}(t)$ denotes the $1 \times (b + a)$ vector containing the values $w_i(t)$ for all the transient and absorbing states, then the inventory over time can be computed by iteratively applying the transition probability matrix to $\boldsymbol{W}(t)$ and adding the external contribution of the constituent to each reservoir:

$$\boldsymbol{W}(t) = \boldsymbol{W}(t-1)\mathbf{P} + \boldsymbol{L}(t), \tag{20}$$

where the ith entry in the vector $\boldsymbol{L}(t)$ is the amount of the constituent that entered reservoir i from upstream and lateral (i.e., nonvalley floor) sources during the time increment between $t - 1$ and t. The entries in $\boldsymbol{L}(t)$ can be computed by partitioning the influx of constituents according to the distribution of deposition probabilities downstream of the source area, using equations (5)–(8) as described earlier (see eq. [17] for an example):

$$L_i(t) = \boldsymbol{P}_{\text{ext}}(i) \times k(t), \tag{21}$$

where $L_i(t)$ is the ith entry in $\boldsymbol{L}(t)$, $\boldsymbol{P}_{\text{ext}}(i)$ is the ith entry in $\boldsymbol{P}_{\text{ext}}$, and $k(t)$ is the time-varying influx of the constituent into the reach from the external source.

If the constituent decomposes appreciably with time as a result of physical, chemical, or biological processes, this decomposition can be accounted for after each time increment. For example, in the case of radioactive decay, the rate of decay is proportional to the amount of the substance present. Then the concentration $\chi_i(t)$ of an unstable constituent in state i at time t is

$$\chi_i(t) = \chi_i'(t)e^{-\lambda\tau}, \tag{22}$$

where λ is the radioactive decay constant for the substance, $\chi_i'(t)$ is its concentration on sediment at time t before accounting for decay, and τ is the length of the time increment. For other types of decomposition processes, appropriate relationships must be substituted for equation (22). The inventory at time t is computed by substituting $\chi_i(t)$ for

$c_i(t)$ in equation (19), and its redistribution in time increment $t + 1$ is given by equation (20). Then decay can be recomputed after the next time increment with equation (22) or an appropriate decay function.

To illustrate how equations (19) and (20) can be used to track the redistribution of sediment-bound constituents, assume for simplicity that a chemically stable contaminant enters the valley bound to sediment over a period of 30 yr (fig. 6*a*); examples of such contaminants could be heavy metals, long-lived radionuclides such as plutonium, or organic pollutants such as polychlorinated biphenyls. In this example, the concentration of the contaminant on sediment entering from upstream changes over time, but the sediment budget is not affected.

Figure 6*a*, computed iteratively using equation (20), shows how flood plain storage can moderate the downstream delivery of a contaminant introduced into a river reach from upstream. Equation (20) can also be used to track the amount of the hypothetical contaminant stored in each of the six transient states over time (fig. 6*b*, 6*c*). In the case of the hypothetical valley floor with the sediment budget in table 2, the flood plains are expected to store a much greater proportion of the contaminant than the channel deposits because (1) the rate of sediment exchange with the flood plain is greater than the rate of exchange with the channel (table 2), leading to a larger amount of the contaminant entering the flood plain, and (2) the residence time of sediment in the flood plain is greater than that for channel-stored sediment. The model also predicts the spatial distribution of contaminant storage over time: given the sediment budget, the flood plain in reach A should contain nearly 75% more contaminant than the flood plain in reach C at peak inventory but almost 50% less after 100 yr. Such a capability could be useful for predicting potential risks to riparian ecosystems posed by contaminated sediment introduced from anthropogenic sources or environmental disasters, for identifying potential problem areas, and for estimating the timescales required for natural decontamination of fluvial systems.

Even after contaminated sediment releases cease, active fluvial deposits can continue to pose downstream risks to human health and ecosystems as they erode. The framework presented here can be used to assess the magnitude and relative influence of different sediment sources on the downstream contaminant flux. If there is only one absorbing state, at the downstream boundary of the model, the flux over time $f_i(t)$ (mass/time) of a constituent into the absorbing state from each transient state i is

$$f_i(t) = h_i(t) \times \chi_i(t), \qquad (23)$$

while the total flux of the constituent past the downstream model boundary is

$$F(t) = \boldsymbol{L}_x(t) + \sum_{i=1}^{b} f_i(t), \qquad (24)$$

where $\boldsymbol{L}_x(t)$ is the last entry in $\boldsymbol{L}(t)$, the mass of the constituent supplied to the system from external sources during time t that immediately enters the absorbing state (obtained from eq. [21]).

Figure 7 demonstrates how the theory can be used to compare the relative and absolute magnitudes of future sources of downstream contamination. The calculations in figure 7 begin in year 30, when the influx of the contaminant from external sources has ceased, after which valley-stored sediment is the only source of contamination at the downstream boundary. The initial condition for the calculations in figure 7 is the distribution of contaminant inventory in the transient states in year 30 (fig. 6*b*, 6*c*). The subsequent contaminant flux into the absorbing state, which originated from each of the six transient states in year 30, was computed using equations (15) and (23). The heavy solid lines in figure 7 show the total flux of the contaminant at the downstream boundary after year 30 (computed using eq. [24], with $\boldsymbol{L}_x = 0$). Figure 7*a* compares channel sources with flood plain sources over time and predicts that the contribution of the contaminant from initially channel-stored sediment will decrease rapidly after the releases from upstream stop; the contaminant flux at the downstream boundary after year 30 will be dominated by sediment stored in the flood plain in year 30. This is probably a reasonable statement for many contaminated rivers, making the estimate of the amount initially stored in the flood plain critical, whether it is accomplished through computation, as in equations (19)–(22), or by direct field inventory (e.g., Marron 1992). Figure 7*b* compares future contaminant sources spatially, showing that the greatest source of the downstream contaminant flux will initially be reach C (the downstream reach). As this reservoir is depleted, upstream sources (reaches A and B) become more important, and by year 50, the dominant source of contamination will be sediment that was stored in reach A on cessation of the releases.

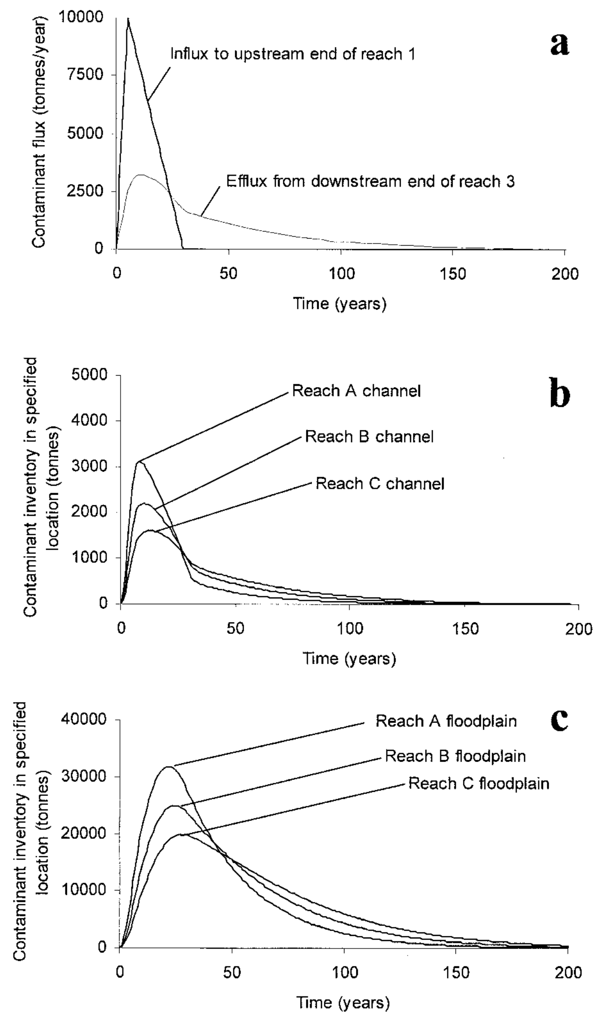

Figure 6. Fate of sediment-bound contaminants in the hypothetical valley floor. *a*, Flux of hypothetical contaminant over time into and out of modeled reaches. Contaminant concentration on sediment entering reach A assumed to increase to a maximum of 5 mg/g sediment (i.e., 10,000 T/yr) after 5 yr and then decrease to 0 after 30 yr. Modeled efflux at the downstream boundary (transport into the absorbing state) accounts for exchanges with sediment deposits in the valley floor and demonstrates the role of sediment storage in moderating the flux of contamination downstream. *b*, Inventory of hypothetical contaminant over time within the three channel reservoirs, given the influx indicated in *a*. *c*, Inventory of hypothetical contaminant over time within the three flood plain reservoirs.

The calculations illustrated in figure 7 can provide guidance in designing mitigation for contaminated sediment. For example, if a mitigation strategy entailed excavating contaminated sediment from the flood plain beginning in year 30, when the releases ended, it would be possible to compare the relative effects over time of excavating in different reaches by changing the initial distribution of contamination. Traditional sediment routing models, which compute changes in storage along channel reaches, cannot predict the fate of sediment in flood plains. Since much of the contaminated sediment along the world's rivers resides in flood plain deposits, the stochastic model of sediment trajectories provides a tool for supporting management decisions in an increasingly important area of environmental concern.

Discussion

In this article, we have presented a theoretical framework for analyzing the trajectories of sediment and associated chemical constituents in alluvial valley floors in steady state. The approach incorporates the stochastic nature of sediment movement through alluvial valleys, which contain fluvial deposits of varying mobility. The model can provide useful information for some valleys over appropriate timescales. However, the current version of the theory simplifies or ignores aspects of sediment routing that are important in many valley floors, including (1) selective transport and deposition as a result of particle size sorting; (2) transient conditions, in which significant changes in sediment storage over decades or longer cause systematic changes in the transition probabilities; and (3) the stochastic nature of forcing mechanisms that govern the sediment budget of the valley floor. We elaborate on each of these limitations and discuss ideas for adapting probability theory to valleys where these factors are significant.

Multiple Particle Size Classes. In many rivers, the fate of sediment delivered to streams is largely determined by particle size. This effect is particularly important in gravel-bed rivers, where particle size determines the mechanism by which sediment is transported and the types of the deposits in which it can be stored. Coarse particles such as gravel are usually only exchanged with the channel bed. Fine particles, including sand, silt, and clay, generally travel in suspension in the water column and are more likely to interact with the flood plain. Treating all particles equally fails to capture mechanisms that are dominant in some rivers. Furthermore, many important environmental issues in rivers relate to the particle size–dependent behavior of sediment and its influence on fluvial and riparian ecosystems. For example, deposition of fine sediment in gravel-bed channels can smother spawning gravel (Cordone and Kelly 1961; Lisle 1989) and

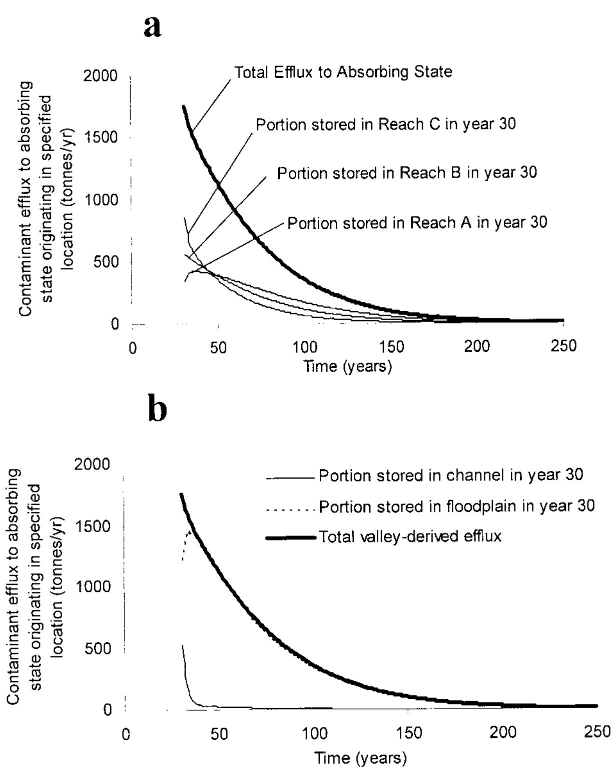

Figure 7. Comparison of modeled "future" contaminant sources at the downstream boundary (into the absorbing state). Initial condition is contaminant distribution in year 30, when contaminant influx from upstream ceases. Heavier lines in *a* and *b* indicate total computed flux of contaminant into the absorbing state. *a*, Comparison of contaminant contributions from sediment stored in reaches A, B, and C in year 30. *b*, Comparison of contaminant contributions from sediment located in the channel and in the flood plain in year 30.

rearing habitat in pools and inhibit food production from riffles. Also, particle-bound contaminants preferentially adhere to fine-grained sediment, which is stored in large quantities within flood plain deposits (e.g., Marron 1992).

One relatively simple way to incorporate this effect would be to subdivide sediment storage reservoirs by particle size. Each particle size fraction could be treated as a distinct transient state, with characteristic probabilities for erosion and subsequent deposition. In this way, one could account for differential transport in mixed-load river channels. This approach could also be used to simulate particle abrasion, where it is significant, by allowing sediment transfer among transient states that represent particle size classes.

Another approach for simulating particle size sorting in rivers is to compute separate sediment budgets and transition matrices for at least two particle size classes. The classes could be the bed material load, which interacts with the bed, and the wash load, which is generally not found in the bed but can interact with the flood plain. By estimating sediment budgets for each of these two classes, one could compute separate transition matrices for bed material load and for wash load. Then the analyses presented could be performed for both classes. This approach was adopted for simulating two particle size fractions in upper Los Alamos Canyon, New Mexico (Malmon 2002).

Transient Case. The steady state model presented here incorporates an important assumption, namely that the transition probability matrix remains constant over time. The basic calculations in equations (4)–(8) assume that sediment fluxes and the amount of sediment in storage (m_i) are constant when averaged over multiyear time increments. Fortunately, the masses of most alluvial deposits are so large that they change only slowly with imbalances in their sediment budgets. However, non–steady state conditions affect the erosion probabilities in at least three ways: (1) changing masses of sediment reservoirs lead to time-varying erosion probabilities (through the denominator in eq. [4]); (2) storage changes may alter channel geometry (e.g., changes in channel conveyance capacity or bank height, transitions from single-thread to braided) sufficiently to affect rates of sediment exchange processes such as flood plain sedimentation and erosion, thereby affecting the deposition probabilities (eqq. [5]–[8]); and (3) temporal and spatial variations in downstream sediment flux also affect deposition probabilities (eqq. [5]–[8]).

There are many applications in which it would be useful to route sediment through valley floors containing reservoirs that dynamically adjust to sediment supply and transport capacity. Some large changes in the sediment budgets of valley floors appear to be related to climate and flow changes (e.g., Schumm and Lichty 1961). Fluctuations in the amount of sediment stored in some alluvial valleys might be an inherent property of fluvial systems in semiarid regions (e.g., Schumm and Hadley 1957). In upland catchments in both humid and arid regions, transient sediment storage is driven by spatial and temporal variability in sediment supply (e.g., Benda and Dunne 1997*b*). Anthropogenic perturbations such as dam construction (Williams and Wolman 1984) and removal can produce transient conditions in large river valleys. Non–steady state conditions can also be instigated by natural but infrequent events, such as extreme climatic events or fires.

In the case in which a large, discrete sediment input influences the sediment budget temporarily, the fate of the input might be determined with an offline calculation and the subsequent state of the valley floor treated as a steady state using the equations presented. For example, it may be of interest to estimate the fate and residence time of particles introduced to the fluvial system from a large landslide or anthropogenic input (e.g., a spill from a mine tailings pond or dam). Such a perturbation could temporarily affect sediment fluxes or rates of exchange, but after the perturbation the fluvial system might be considered to be in an approximate steady state with respect to sediment storage.

In some valleys, transient conditions can persist, causing feedbacks between sediment storage and valley geometry, which would cause components of the sediment budget to continue to change over time. In this situation, the transient case can be modeled by changing the entries in the transition matrix according to observed or predicted changes in the sediment budget. The Chapman-Kolmogorov equation (eq. [12]) does not apply in this case, but the same results can be achieved numerically using a time-varying transition matrix. Thus, the main task is to improve the scientific basis for quantifying changes in erosion and deposition rates in response to changes in sediment storage.

This topic has recently attracted research interest. Lisle and Church (2000) proposed that the sediment transport capacity of alluvial sediment storage reservoirs is a unique positive function of storage volume. For example, the transport rate of sediment through the channel bed is significantly affected by channel gradient and particle size, both of which can adjust to changes in sediment storage (e.g., Dietrich et al. 1989). Transport storage functions for alluvial reservoirs, constrained in field settings, could be used to model how the erosion probability in equation (4) would change over time in non–steady state river valleys.

Quantitative relationships between sediment transport, storage, and deposition in alluvial reservoirs are needed to route particles through valley floors undergoing geomorphic changes. An important area of research is to constrain such relationships in field settings. It will be relatively easy to incorporate these relationships into the probabilistic context proposed here.

Stochastic Nature of Forcing Mechanisms. Even in the absence of long-term trends, the forcing mechanisms of fluvial systems are characterized by significant temporal variability (Benda and Dunne 1997*a*). The off-diagonal transition probabilities in equation (9) are likely to be larger during wet years than during dry years. The model presented simulates sediment trajectories using a long-term average sediment budget. This neglects the role of interannual variability and extreme events or of hydrologic persistence, all of which may be significant factors controlling sediment redistribution in valleys.

Statistical methods referred to as hidden Markov models (Baum and Petrie 1966; Baum and Egon 1967; Rabiner 1989) may offer a theoretical framework for incorporating this sort of variability into probabilistic modeling of particle trajectories. Hidden Markov models (HMMs) are probabilistic functions of Markov chains, in other words, models in which the transition probabilities themselves are random variables. These models have been used primarily for applications relating to computer speech recognition, but they could potentially be useful for generating random particle trajectories in valleys driven by stochastic external forcings. Rabiner (1989) provides an excellent overview of the theory intended for researchers outside the field of mathematics.

To apply HMMs to model particle trajectories in an alluvial valley floor, the model would be specified by the following: (1) a finite number of hidden or unknown states, corresponding to different event magnitudes (e.g., wet or dry years or small and large events); (2) the number of observation symbols in each state, corresponding to the number of sediment storage reservoirs accessible during each flow event; (3) the transition probabilities among hidden states (i.e., the probability of each event occurring); and (4) the transition probabilities among observation symbols or sediment storage reservoirs, given each event has occurred (equivalent to those derived from eqq. [1]–[8] using the sediment budget). Given these four pieces of information, the HMM can be used to generate random particle trajectories through the valley following the steps outlined by Rabiner (1989). The statistical properties of the trajectories generated in this way could be analyzed to compute sediment residence times, the locus and duration of intermediate sediment storage, and the timing of delivery of sediment from alluvial valley floors. The probabilities of the hidden states could be adjusted to reflect longer-term changes in climatic or watershed characteristics. Such applications could be useful for examining the role of events of different magnitudes in the long-term migration of sediment through valleys. Also, the transition probabilities among observation symbols could vary through time to simulate valley floors undergoing major changes in sediment storage or channel geometry.

The primary limitation to applying more sophisticated and realistic probability models to landscapes is the lack of sufficient field data or geomorphic process models with which to estimate the terms in the sediment budget. Advances in both theory and measurement technology are making both of these more available for many field sites. As the basis for modeling and measuring the sediment budget improves, it should be relatively simple to incorporate them into this stochastic framework.

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